Track reconstruction in high density environment

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Abstract

The extension of barrel track finding and fitting algorithm for ALICE Transition Radiation Detector (TRD) [2] based on the Kalman-filtering [3] is presented. The filtering algorithm is able to cope with non-Gaussian noise and ambiguous measurements in high-density environments. The approach have been implemented within the ALICE simulation/reconstruction framework (AliRoot)[4], and algorithm efficiency have been estimated using the AliRoot Monte Carlo data.

TRD-GENERAL DESCRIPTION AND WORKING PRINCIPLE

The ALICE TRD consists of 540 chambers surrounding the Time Projection Chamber (TPC) in six layers with an overall length of about 7 m. The total sensitive area is roughly 750 m²; the largest chamber is 159 cm long and 120 cm wide. Each module is about 13 cm thick, including radiator, electronics and cooling. The total anticipated radiation thickness for six layers is about $0.15X_0$.

A schematic cross section of a TRD module is shown in Fig. 1. The gas volume is subdivided into a 3 cm drift region and a 0.7 cm amplification region, separated by a cathode wire grid with 0.25 cm wire pitch and 75 μ m wire diameter. The anode wires have 0.5 cm pitch and 20 μ m diameter. The drift chambers are equipped with cathode pads of varying sizes¹. The whole system will consist of about 1.18 million channels (readout pads). The maximum drift time is about 2 μ s and the induced signal is sampled on all channels at 10 MHz to record the time evolution of the signal [7, 8]. A typical signal generated by a particle track through a prototype drift chamber is also shown in Fig. 1.

A 4.8 cm thick radiator is placed in front of each gas volume. Transition Radiation (TR) is emitted by particles traversing the radiator with a velocity larger than a certain threshold [6], which for typical materials corresponds to a Lorentz factor of $\gamma \approx 1000$. The produced TR photons have energies in the X-ray range (1 to 30 keV) and a high-Z gas mixture (Xe, CO₂ (15%)) is used to provide efficient absorption of these photons.

TRD TRACKING

The TRD reconstruction is an integral part of the overall Alice tracking and is based on the Kalman–filter algorithm. In addition, fast linear Riemann sphere fit for TRD stand– alone tracking and seeding of secondary particles was implemented.

The main challenges of tracking in the Alice TRD are following:

- Significant material budget in the TRD volume. Fraction of tracks absorbed in the material is about 35% of all tracks. Mean energy loss is about 15% of particle energy.
- In the expected high density environment, clusters from different tracks may overlap; therefore, a certain number of clusters become merged, and others may be significantly displaced. Moreover, in the propagation road of the track, defined by track position uncertainty, on average there are about 1.5 clusters $(dN_{ch}/d\eta=5000)$.

In AliRoot framework, ROOT Geometrical Modeler [9] is used to get information necessary for the calculation of



Figure 1: Schematic view of a TRD chamber (not to scale). The left cross section shows a projection of the chamber in the x-z plane, perpendicular to the wires, the right one shows a projection in the x-y plane, which is the bending plane of the particles in the ALICE magnetic field. A particle trajectory is also sketched. The insert shows the pulse height versus drift time on eight cathode pads for an example event. One time bin corresponds to 100 ns.

¹The width of the pads ranges from 0.664 to 0.818 cm (y direction), their length from 7.5 to 9 cm. (z-direction)

energy loss and multiple scattering. The mean query time to obtain information about local material density, radiation length, Z, A is about 15 μ s. Two options were considered:

- Propagate track up to material boundary defined by the modeler. Get local material parameters and propagate the track using current material parameters.
- Calculate mean parameters between start and end points of the tracklet (density, density. Z/A, radiation length)

The second option is much faster than the first (only one track propagation), moreover it is reusable in case of close parallel track hypothesis.

The standard Kalman approach is not efficient enough in high density environment. Significantly better results can be achieved using Combinatorial Kalman filter [5]. However, the usage of Combinatorial Kalman filter was abandoned because of huge combinatorics and consequent CPU requirements. The fast, robust search for the tracklet² is described in section - Tracklet Search.

Local reconstruction

The TRD is a three dimensional detector. The xcoordinate of the space point is given by the drift time (time bin). In the first approximation, the drift time can be translated to a position (distance from the anode wire plane) if the drift velocity is known. In the TRD v_D is constant in a large fraction of the detector ($v_D = v_0 = 1.5 \text{ cm}/\mu \text{s}$ in the drift region), but (in general) higher in the amplification region. We use the following linear approximation:

$$x \approx v_{\rm D}^{av} t_{\rm D} ,$$
 (1)

where $v_{\rm D}^{av}$ is an average drift velocity.

The drift time t_D depends strongly on the distance to the anode wire (z'). Electrons drifting at maximal $z' \approx$ 0.25 cm have a longer drift path and – on top of that – cross the low field region between two anode wires. Consequently, we observe a drift time offset depending on z'. For electrons coming from the drift cathode $(x \approx 3.35 \text{ cm}, z' \approx$ 0.25 cm) it is around 120 ns as compared to the value at $(x \approx 3.35, z' \approx 0 \text{ cm})$. In the following this effect is referred to as unisochronity effect.

The most crucial for the performance of the ALICE TRD is the position reconstruction in the bending plane (y-direction) of the particles in the ALICE magnetic field. This defines the transverse momentum resolution of the TRD. The *y*-coordinate is reconstructed from the pulse-height distribution on adjacent pads. Lookup table of mean position as a function of amplitudes in three adjacent pads is used instead of COG to minimize non-linearity. The main advantage of the lookup table approach is that it is as fast as the COG method, and in addition provides results with better precision than fit with Mathieson function. The shape of the signal in the pad direction (Pad Response Function–PRF) depends on the inclination angle of

the track. The lookup table was optimized for high momentum tracks with small inclination angle.

In the third dimension (z-direction), parallel to the magnetic field, the resolution is limited by pad size as the width of PRF in z-direction is negligible with pad size. A tilted-pad design is employed to increase the tracking capabilities in this direction.

The signals that are read out from the cathode pads are induced by the positive ions generated in the electron avalanches near the anode wires. Since the massive ions move slowly compared to the electrons, the signals exhibit long tails. Convolution with the response of the PASA yields the Time Response Function (TRF), which is asymmetric.

TRF gives rise to a strong correlation between the signal amplitude in subsequent time bins. These correlations affect especially tracks with big angular inclination (angular effect). A way to minimize this effect is to remove the tails from the data by deconvolution - tail cancellation. The tail cancellation is performed on the raw data before the clusterization algorithm.

The signal unfolding is performed for clusters with extended shape. However, the precision of position measurement of unfolded clusters is significantly worse than in the case of isolated clusters ($\approx 2 \text{ mm resp. } \approx 0.4 \text{ mm}$).

TRD stand-alone tracking

In order to to minimize CPU requirements, the linear Riemann sphere fitting is used to fit TRD tracks in stand– alone tracking:

$$(x - x_0)^2 + (y - y_0)^2 = R^2 = >$$
 (2)

$$a + u.b + t.c + v = 0$$
 (3)

$$a = 1/y_0 \tag{4}$$

$$b = x_0 / y_0 \tag{5}$$
$$c = R^2 - x_0^2 - y_0^2 \tag{6}$$

$$t = \frac{1}{2}$$
(7)

$$u = \frac{2 \cdot x}{x^2 + y^2}$$
(8)

$$v = \frac{2.y}{x^2 + y^2} \tag{9}$$

(10)

The vertex constraint can be applied fixing c parameter to 0.

In the TRD chamber the y coordinate is not measured directly, the pads are tilted. Instead of y coordinate, w coordinate is measured:

$$w = y - h_t (z_c - z_t)$$
 (11)

here z_c is the center of the pad in z direction, z_t is the z position of the track and h_t is the tangent of the tilting angle.

In the following we assume that the track z position depends linearly on x (on the range of about 60 cm) and

²Tracklet is set of clusters belonging to the same track in one chamber.

 $x^2 + y^2 \approx x^2 + w^2$. Under this assumptions the original formula 10 is modified to the following:

$$a + u.b + t.c + v + d.2.h_t + e.2.h.(x - x_r).t - 2.(w + h_t.z_c).t = 0$$
(12)

t

$$=\frac{1}{x^2 + w^2}$$
(13)

$$u = \frac{2 \cdot x}{x^2 + w^2}$$
(14)
$$e = \frac{dz}{dx}$$
(15)

where *d* is the position of the track at the reference point x_r . The described linear fitting algorithm gives results similar to non–linear fit, but is significantly faster (≈ 10 times).

Tracklet Search

Several possibilities of association of the clusters to the track in the high flux environment were investigated. The straightforward use if the nearest cluster to the expected track prolongation is not sufficient. In the case of TRD tracking in high flux environment, on average about 1.5 clusters can be associated to each track at each propagation layer. In our approach the straightforward cluster association was replaced with tracklet search in each plane.

We tried to reduce possible combinatorics at a very early stage, before CPU expensive calculations, by using the knowledge of the specific topology of TRD detector. The main assumptions are following:

- The mean $y (r \phi)$ resolution of the tracklet position measurement is on the level of 0.4 mm. The resolution of the track extrapolation is about 2 times worse than the tracklet resolution. All clusters inside one plane are systematically shifted in x-direction because of the unisochronity effect. It is impossible to correct for this shift unless the position resolution in the zmeasurement is precise enough to determine the distance of the electrons to the wire.
- Z error distribution of the clusters is described by rectangular distribution given by length of pads (±4 cm). Averaging about 20 samples (depending on the sampling frequency) on the same plane does not improve z resolution. The only exception is in the case when the track crosses the boundary between two pad-rows. Probability to cross the pad-row depends on the z-inclination of the track and is on the level of 15%.

$$P_{\rm cross} = \frac{L_{\rm drift} |\tan \theta|}{L_{\rm pad}} \tag{16}$$

The ratio of the drift length (L_{drift}) to the pad length (L_{pad}) is about 0.3. Therefore, tracks with inclination angles $\tan\theta$ smaller than 3 can not cross the pad-row more than once. Tracklet with more than one change can be excluded from the consideration. In addition

the tracklets with the changing of z-direction in opposite direction in respect with the track direction are also forbidden.

Applying previous assumptions, the search for a tracklet in 3 dimensions can be reduced to a few (number of time bins) searches in two dimensions (x - y). Even such reduction of combinatorial problem is not sufficient for usage of the Combinatorial Kalman filter. Remaining combinatorics in case of two close tracks can reach of about 2^{20} combinations. Two alternative linear algorithms were implemented inside reconstruction framework:

- Iterative algorithm
- Projection algorithm

In the first approximation of the iterative algorithm, clusters most close to the track are taken and trivial z swapped clusters are removed. Then the iteration step is performed:

- Tracklet position, angle and their uncertainty are calculated
- Updated (weighted) mean position is calculated (track+ tracklet)
- χ^2 -calculation for the tracklet
- Clusters closest to the weighted mean are taken

The projection algorithm is similar to the Least Trimmed Squares regression (LTS) in 1 dimension. Here we define outliers as atypical infrequent observations; data points which do not appear to follow the characteristic distribution of the rest of the data (see Fig.2). These outliers are removed from the the sample of the data.

The algorithm is following:

- Loop over possible change of z direction
 - Calculates residuals.
 - Find sub-sample (number of time bins in plane) of clusters with minimal χ^2 distance to the weighted mean (track + tracklet). Simple sort used $N \log(N)$ problem.

Precision of local tracklet reconstruction

The precision of the tracklet measurement is determined mainly by the angular effect and by the non-isochronity effect in combination with the Landau fluctuation of the energy deposition:

$$\sigma_y = \sqrt{\sigma_0^2 + \tan^2 \phi k_0^2} \tag{17}$$

For non zero inclination angles, the contribution of the electronic noise to the precision of measurement ($\sigma_0 \approx 0.2 \text{ mm}$) is negligible compared to the angular term. The factor k_0 is determined as a linear fit parameter and corresponds to the width of the time response function



Figure 2: Four different views of tracklet. 1.) Pad response function. 2.) Residuals as function of time bin. 3.) Projection of residuals. 4.) Originating data samples over threshold. Top 4 pictures: views for non overlapped tracklet. Bottom: views for two overlapped clusters in the same z-slice.

convoluted with the uncertainty of the time measurement (unisochronity effect). In the first approximation, the unisochronity is the same for all measurements at given chamber, therefore the tracklet position resolution does not scale with the square root of number of samples. It can be noticed, that the uncertainty of the position measurement does not scale linearly with the shape of the cluster σ_{PBF} :

$$\sigma_{\rm PRF} = \sqrt{\sigma_{\rm PRF0}^2 + (\tan^2 \phi) k_1^2} \tag{18}$$

The fitted factor k_1 is roughly the same as the factor k_0 from the equation 17, but the width of electronic Pad Response function ($\sigma_{PRF0} \approx 3$ mm) is much larger than σ_0 . Therefore, in the Kalman filter and as well in the Riemann fitter, the equation 17 is used to estimate the tracklet position uncertainty. This solution made a big impact on the achieved momentum resolution. In the case of usage of scaled pad response function width as the position uncertainty estimate, the σ_0 of uncertainty is overestimated for tracklets with small inclination angle and, on the other hand, is underestimated for the low momenta tracks with big inclination angle. However, in the high density environment, the precision of the cluster position determination is affected by the presence of overlapped tracks. The position resolution for a cluster in the overlap region is much worse than for isolated tracks. These effect can be seen in the Fig. 2 (comparing upper (isolated tracklet) and lower part (overlapped tracklets). The shape of the cluster can indicate the such overlap. Other variable, which indicates the degree of the cluster position uncertainty is the intrinsic resolution σ_{in} of the position measurement inside of one TRD plane:

$$\sigma_{\rm in}^2 = \frac{\sum^{N_{\rm tbins}} \mathrm{d}y_i^2}{N_{\rm tbins} - 1} \tag{19}$$

The error estimate of the tracklet position uncertainty is modified to the following form

$$\sigma_y^2 = (\sigma_0^2 + (\tan^2 \phi).k_0^2) + \sigma_{\rm in}^2/N_{\rm tbins}$$
(20)

where first two terms correspond to effective error parameterization (formula 17) and the last term corresponds to local (chamber) intrinsic resolution (19). The formula 17 is interpreted as the lower limit of uncertainty.

CONCLUSION

The fast, robust tracking algorithm, able to cope with ambiguous measurements in high-density environments was implemented in the TRD detector. Using the tracking capabilities of the TRD, the performance of the barrel tracking algorithm in terms of p_t resolution is improved by a factor of $\approx 40\%$ in comparison with setup without the TRD detector [10]. Using robust projection algorithm, the fake ratio, defined as the number of the wrongly associated tracks to the overall reconstructed tracks in the TRD was decreased (from 10% to 5%) in comparison with previously used (non combinatorial) Kalman filter without significant increase of CPU requirements.

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