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# Theoretical uncertainties in high $p_T$ heavy flavor production and modification

EMMI HF workshop, December 2016

GSI, Darmstadt, Germany

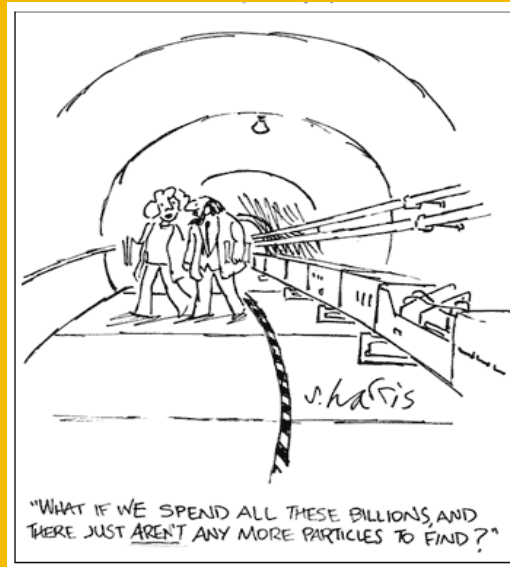
# Plan for the talk

Thanks to the organizer for the opportunity to talk remotely



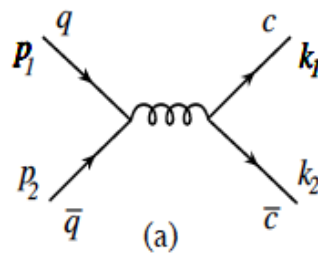
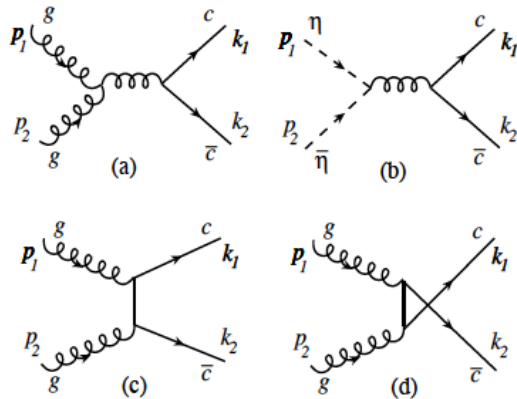
- Production mechanisms for heavy flavor
- Energy loss vs full parton showers
- Sensitivity to medium properties
- Sensitivity to models of the medium
- Uncertainties in quoting  $\hat{q}$
- Non-locality of non-Abelian in-medium parton splitting
- Conclusions

# I. Heavy flavor production mechanisms



# Fixed flavor number scheme

- FFNS at LO very large K factors (4-5)

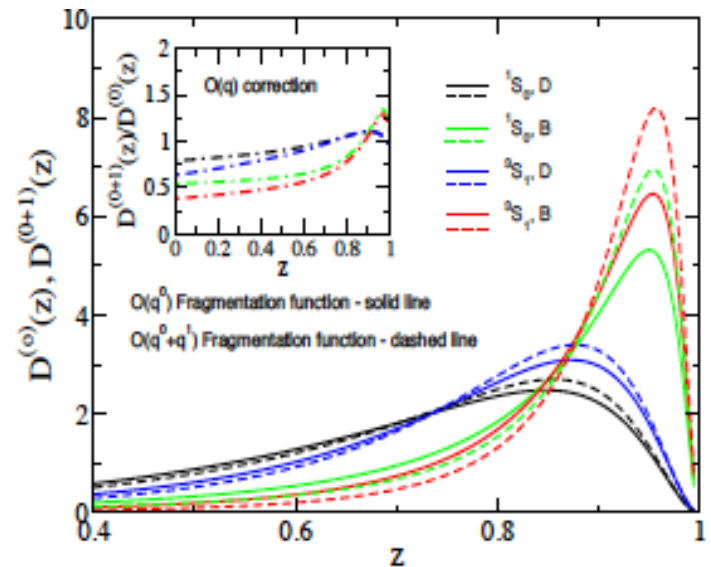
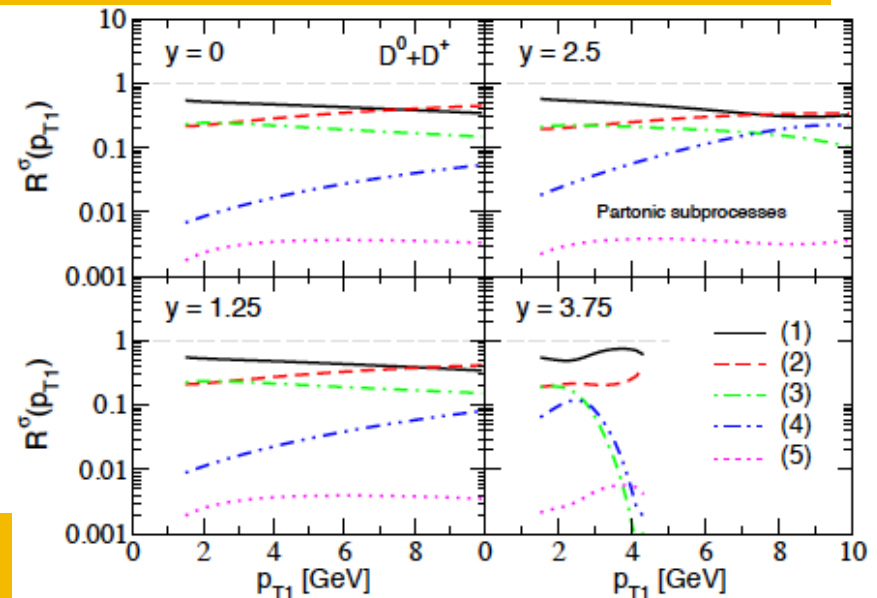
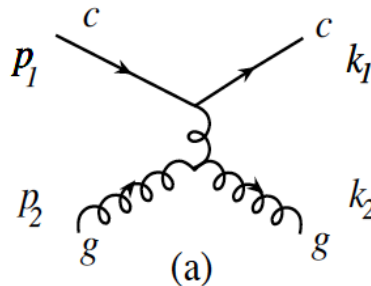


Vitev et al. (2007)

- FONLL – combines NLO with next-to-leading log resummation

Tries to capture the splitting to heavy quarks purely perturbatively

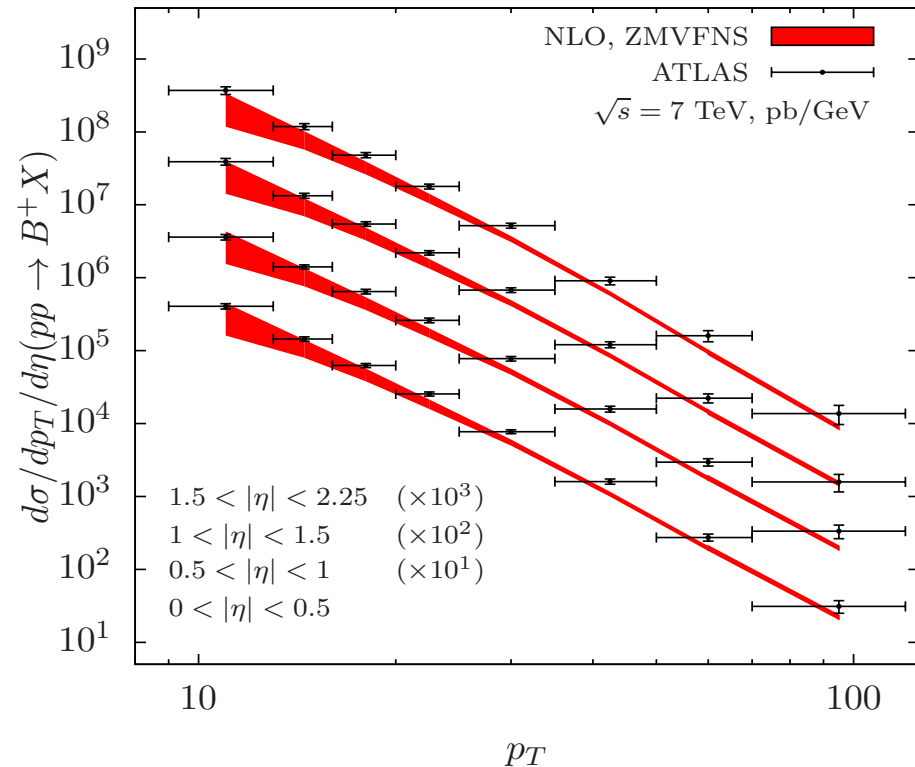
- VFNS at LO reduces or eliminates the K factors



# ZMVFS open heavy flavor at NLO

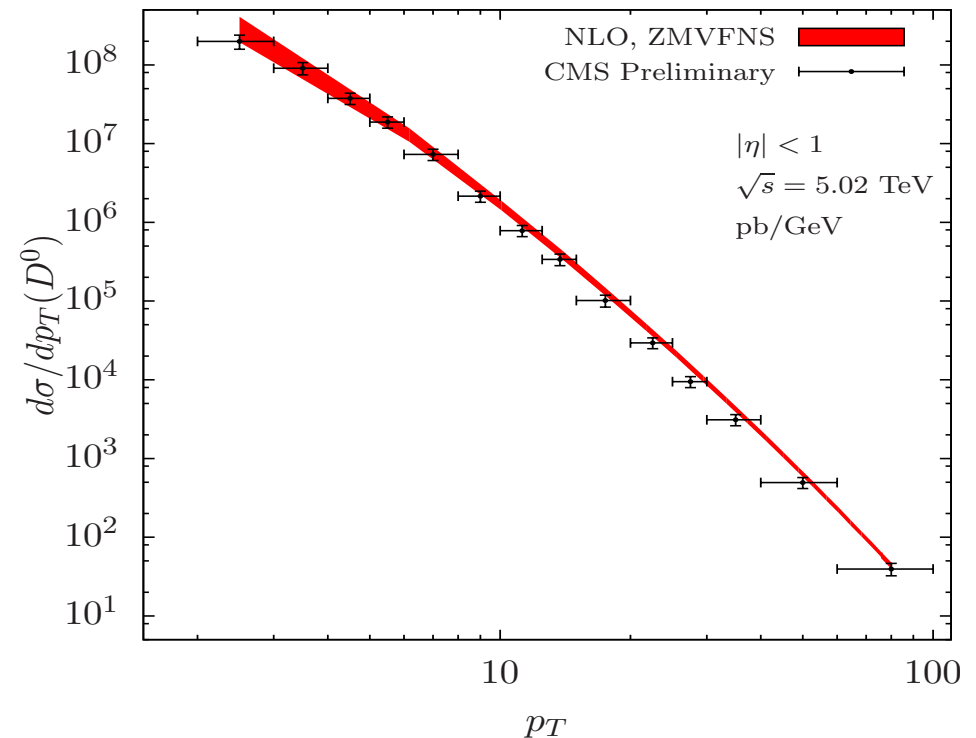
- Perform an NLO calculation

F. Ringer et al. (2016)



Kneesch et al. (2008)

$$\frac{d\sigma_{pp}^H}{dp_T d\eta} = \frac{2p_T}{s} \sum_{a,b,c} \int_{x_a^{\min}}^1 \frac{dx_a}{x_a} f_a(x_a, \mu) \int_{x_b^{\min}}^1 \frac{dx_b}{x_b} f_b(x_b, \mu) \times \int_{z_c^{\min}}^1 \frac{dz_c}{z_c^2} \frac{d\hat{\sigma}_{ab}^c(\hat{s}, \hat{p}_T, \hat{\eta}, \mu)}{dvdz} D_c^H(z_c, \mu),$$

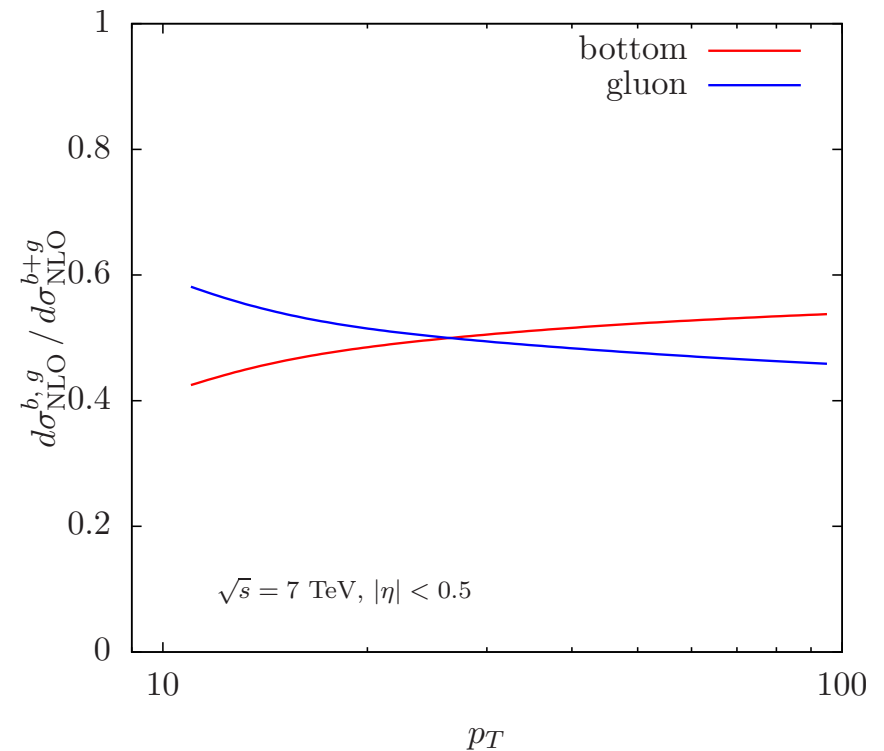
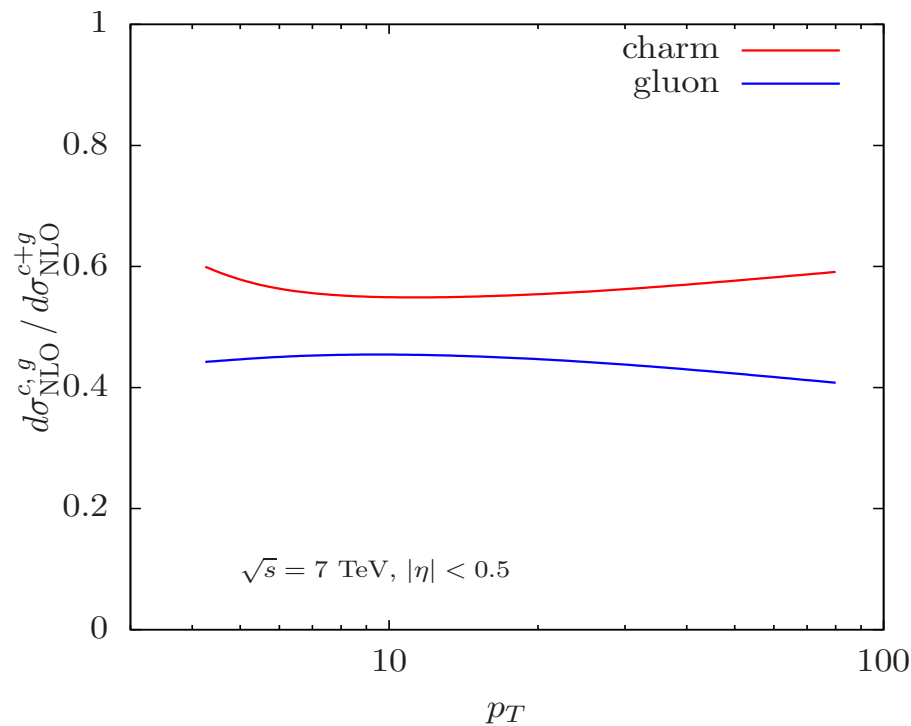


Kniehl et al. (2008)

When  $p_T > m_c, m_b$  Consistent with factorization, non-perturbative physics is long distance

# Implications for heavy flavor modification

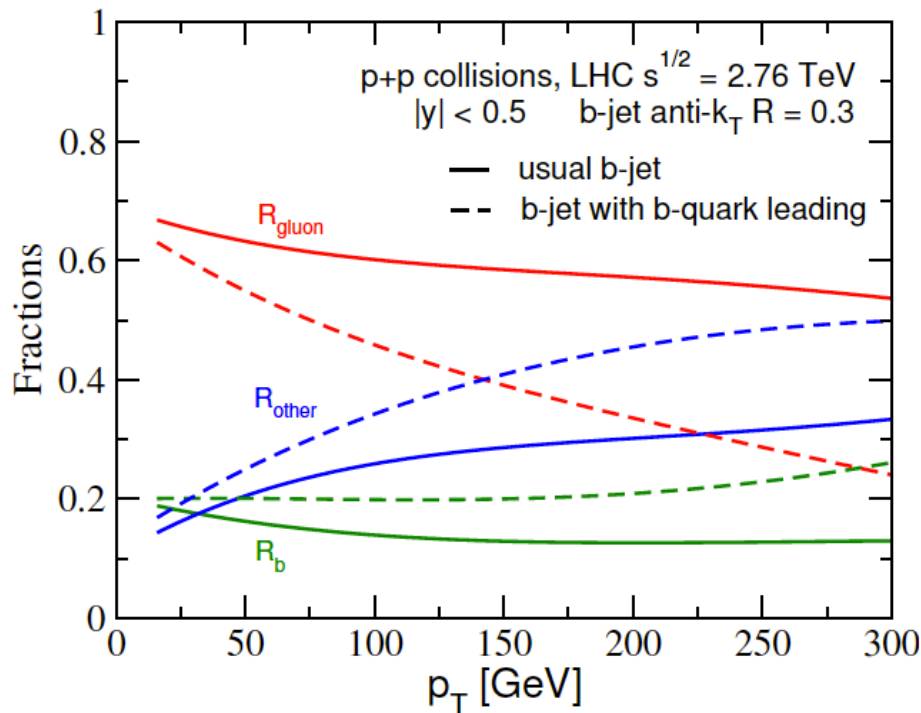
- A very large contribution of gluon FF to heavy flavor



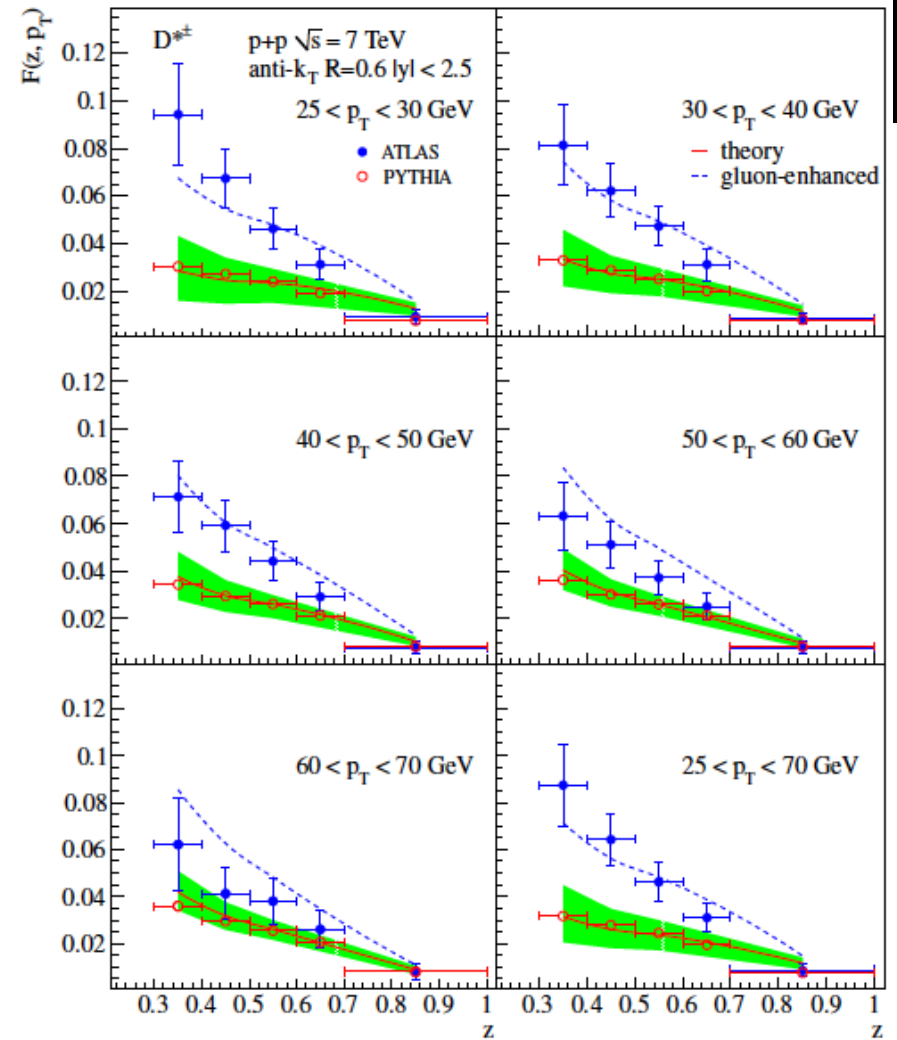
The important implication of this will affect the nuclear modification factor

# The same is true, of course, for b jets, JFF

- Slightly different approach. PYTHIA 8 simulations



Inclusive jets



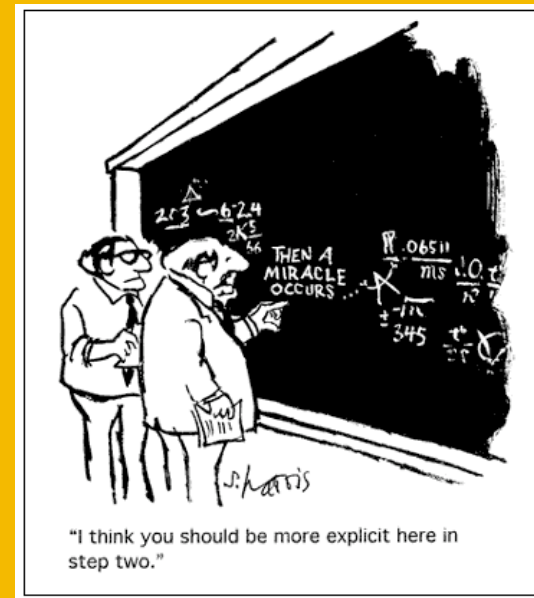
Jet fragmentation functions

- The reason for which b-jets are as suppressed as light jets at high  $p_T$

J. Huang et al. (2013)

Y.T. Chien et al. (2015)

# II. Uncertainties related to the in-medium modification application

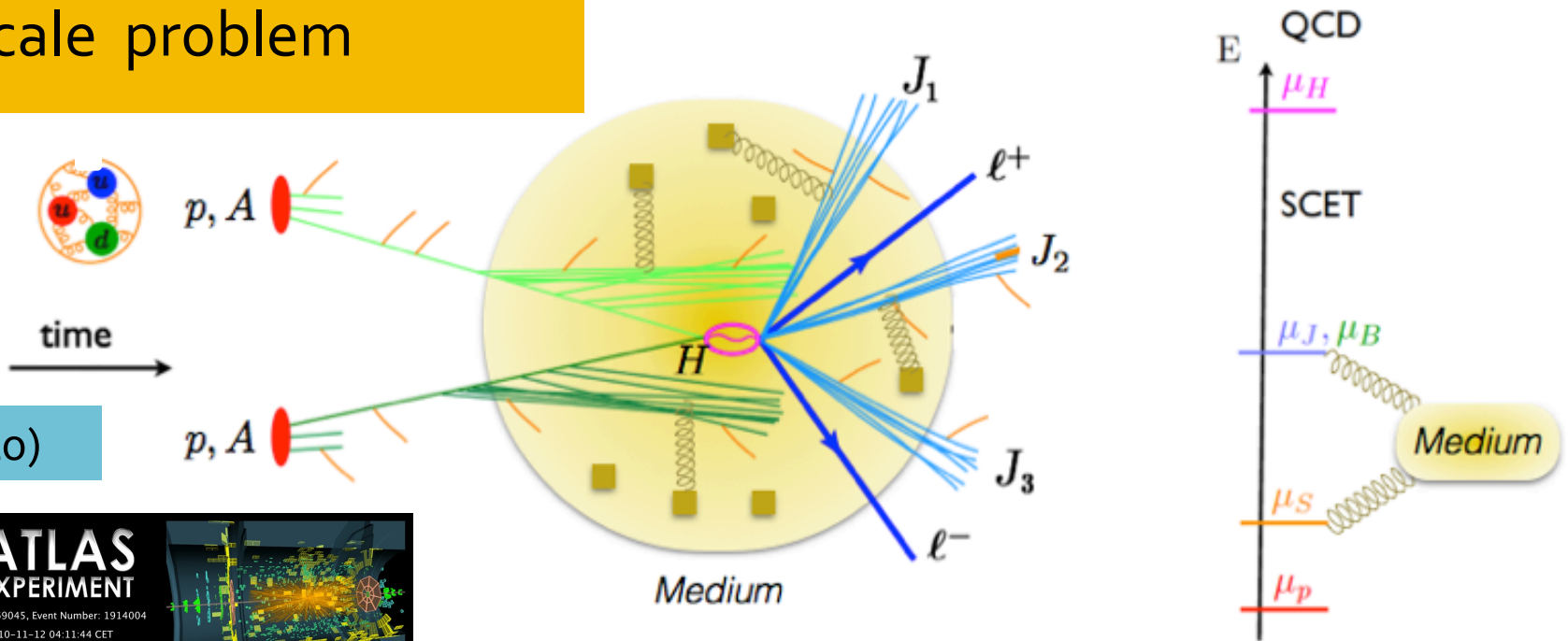




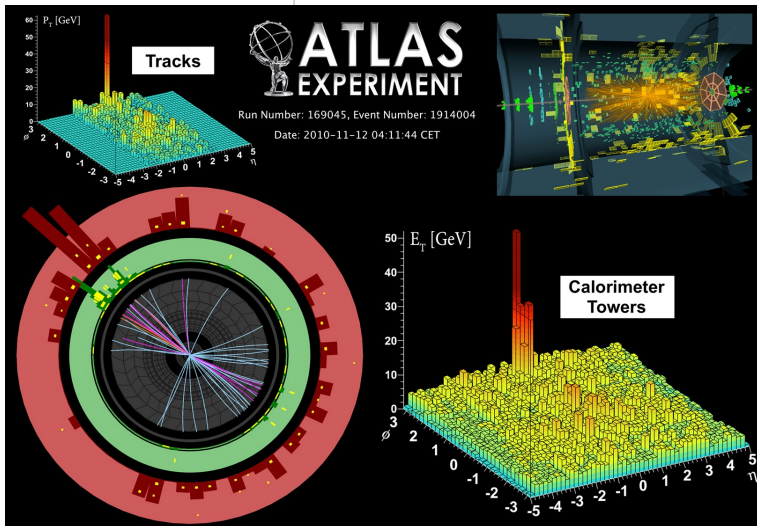
# The big picture (you have seen it before)

- QCD in the medium remains a multiscale problem

Ovanesyan et al. (2011)



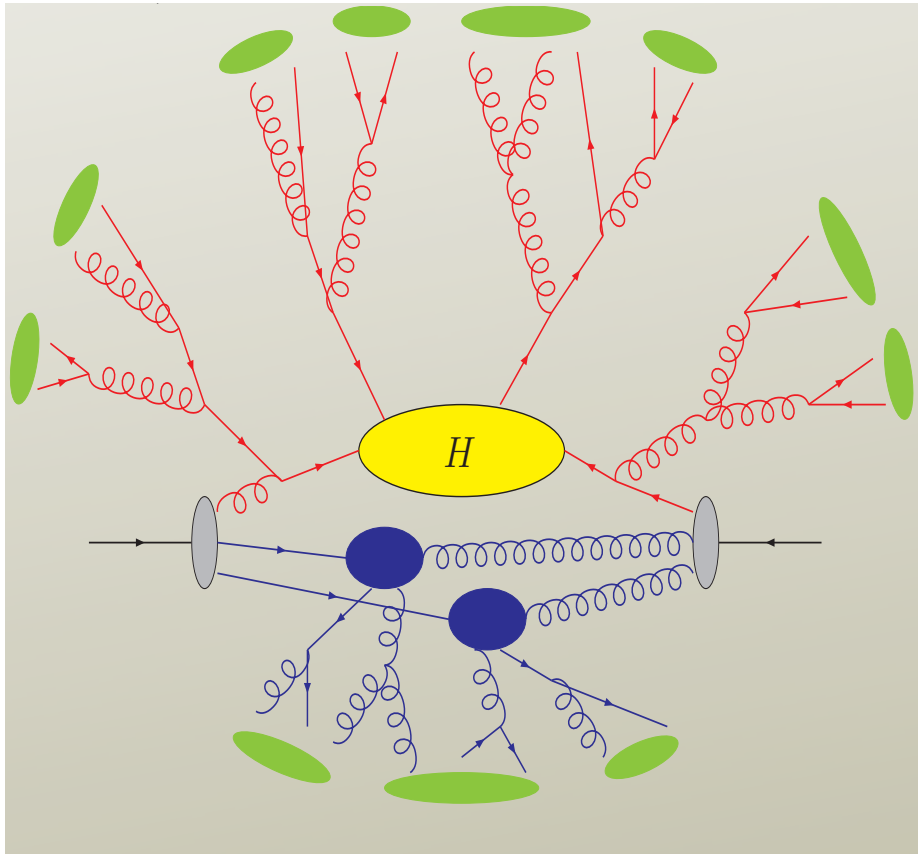
Aad et al. (2010)



- Factorization, with modified  $J$  (jet),  $B$  (beam),  $S$  (soft) functions

$$\sigma = \text{Tr}(HS) \otimes \prod_{i=1}^{n_B} B_i \otimes \prod_{j=1}^N J_j + \text{power corrections}$$

# The splitting kernels



- Splitting functions are related to beam (B) and jet (J) functions in SCET

$$A_{q \rightarrow qg} = \langle J | T \bar{\chi}_n(x_0) e^{iS} | q(\mathbf{p}) g(\mathbf{k}) \rangle$$

$$A_{g \rightarrow q\bar{q}} = \langle J | T \mathcal{B}^{\lambda c}(x_0) e^{iS} | q(\mathbf{p}) \bar{q}(\mathbf{k}) \rangle$$

$$A_{g \rightarrow gg} = \langle J | T \mathcal{B}^{\lambda c}(x_0) e^{iS} | g(\mathbf{p}) g(\mathbf{k}) \rangle$$

$$A_1^{(0)} = \text{Diagram with a blue circle labeled 'J' at position } x_0 \text{, a horizontal line with momentum } p \text{, and a diagonal line with momentum } k \text{ branching off the horizontal line.}$$

$$A_2^{(0)} = \text{Diagram with a blue circle labeled 'J' at position } x_0 \text{, a horizontal line with momentum } p \text{, and a diagonal line with momentum } k \text{ branching off the horizontal line.}$$

$$\Gamma_W^{\alpha,a}(k) = g T_r^a \frac{\bar{n}^\alpha}{k^+ + i\epsilon}$$

Gribov et al. (1972)  
G. Altarelli et al. (1977)  
Y. Dokshitzer (1977)

- In the vacuum we have the DGLAP splitting kernels that factorize from the hard scattering cross section and are process independent

# Heavy quarks in the medium

## Kinematic variables

$$A_{\perp} = k_{\perp}, \quad B_{\perp} = k_{\perp} + xq_{\perp}, \quad C_{\perp} = k_{\perp} - (1-x)q_{\perp}, \quad D_{\perp} = k_{\perp} - q_{\perp}.$$

$$\frac{dN}{dx} \sim \left| \begin{array}{c} \text{Diagram 1} \\ + \\ \text{Diagram 2} \\ + \\ \text{Diagram 3} \end{array} \right|^2 + 2\text{Re} \left[ \begin{array}{c} \text{Diagram 4} \\ + \\ \text{Diagram 5} \\ + \\ \text{Diagram 6} \\ + \\ \text{Diagram 7} \end{array} \right] \times \text{Diagram 8}$$

$$\Omega_1 - \Omega_2 = \frac{B_{\perp}^2 + \nu^2}{p_0^+ x(1-x)}, \quad \Omega_1 - \Omega_3 = \frac{C_{\perp}^2 + \nu^2}{p_0^+ x(1-x)}, \quad \Omega_4 = \frac{A_{\perp}^2 + \nu^2}{p_0^+ x(1-x)},$$

$$\nu = m \quad (g \rightarrow Q\bar{Q}),$$

$$\nu = xm \quad (Q \rightarrow Qg),$$

$$\nu = (1-x)m \quad (Q \rightarrow gQ),$$

F. Ringer et al. (2016)

$$\begin{aligned} \left( \frac{dN^{\text{med}}}{dx d^2k_{\perp}} \right)_{Q \rightarrow Qg} &= \frac{\alpha_s}{2\pi^2} C_F \int \frac{d\Delta z}{\lambda_g(z)} \int d^2q_{\perp} \frac{1}{\sigma_{el}} \frac{d\sigma_{el}^{\text{med}}}{d^2q_{\perp}} \left\{ \left( \frac{1 + (1-x)^2}{x} \right) \left[ \frac{B_{\perp}}{B_{\perp}^2 + \nu^2} \right. \right. \\ &\times \left( \frac{B_{\perp}}{B_{\perp}^2 + \nu^2} - \frac{C_{\perp}}{C_{\perp}^2 + \nu^2} \right) (1 - \cos[(\Omega_1 - \Omega_2)\Delta z]) + \frac{C_{\perp}}{C_{\perp}^2 + \nu^2} \cdot \left( 2 \frac{C_{\perp}}{C_{\perp}^2 + \nu^2} - \frac{A_{\perp}}{A_{\perp}^2 + \nu^2} \right. \\ &- \left. \left. \frac{B_{\perp}}{B_{\perp}^2 + \nu^2} \right) (1 - \cos[(\Omega_1 - \Omega_3)\Delta z]) + \frac{B_{\perp}}{B_{\perp}^2 + \nu^2} \cdot \frac{C_{\perp}}{C_{\perp}^2 + \nu^2} (1 - \cos[(\Omega_2 - \Omega_3)\Delta z]) \right. \\ &+ \frac{A_{\perp}}{A_{\perp}^2 + \nu^2} \cdot \left( \frac{D_{\perp}}{D_{\perp}^2 + \nu^2} - \frac{A_{\perp}}{A_{\perp}^2 + \nu^2} \right) (1 - \cos[\Omega_4\Delta z]) - \frac{A_{\perp}}{A_{\perp}^2 + \nu^2} \cdot \frac{D_{\perp}}{D_{\perp}^2 + \nu^2} (1 - \cos[\Omega_5\Delta z]) \\ &+ \left. \left. \frac{1}{N_c^2} \frac{B_{\perp}}{B_{\perp}^2 + \nu^2} \cdot \left( \frac{A_{\perp}}{A_{\perp}^2 + \nu^2} - \frac{B_{\perp}}{B_{\perp}^2 + \nu^2} \right) (1 - \cos[(\Omega_1 - \Omega_2)\Delta z]) \right] \right\} \\ &+ x^3 m^2 \left[ \frac{1}{B_{\perp}^2 + \nu^2} \cdot \left( \frac{1}{B_{\perp}^2 + \nu^2} - \frac{1}{C_{\perp}^2 + \nu^2} \right) (1 - \cos[(\Omega_1 - \Omega_2)\Delta z]) + \dots \right] \end{aligned}$$

- Full massive in-medium splitting functions now available
- Can be evaluated numerically

# The energy loss limit

3 splitting functions (g to gg is the same)

Dokshitzer et al . (2001)

$$x \left( \frac{dN^{\text{SGA}}}{dx d^2k_{\perp}} \right)_{Q \rightarrow Qg} = \frac{\alpha_s}{\pi^2} C_F \int d\Delta z \frac{1}{\lambda_g(z)} \int d^2q_{\perp} \frac{1}{\sigma_{el}} \frac{d\sigma_{el}^{\text{med}}}{d^2q_{\perp}} \\ \times \frac{2\mathbf{k}_{\perp} \cdot \mathbf{q}_{\perp}}{[k_{\perp}^2 + x^2 m^2][(k_{\perp} - q_{\perp})^2 + x^2 m^2]} \left[ 1 - \cos \frac{(\mathbf{k}_{\perp} - \mathbf{q}_{\perp})^2 + x^2 m^2}{xp_0^+} \Delta z \right]$$

A bit of an ambiguity in the diagonal splitting of how to treat x suppressed terms in the numerator

$$x \left( \frac{dN^{\text{SGA}}}{dx d^2k_{\perp}} \right)_{Q \rightarrow gQ} = \frac{\alpha_s}{\pi^2} C_F \left( \frac{x}{2} \right) \int d\Delta z \frac{1}{\lambda_q(z)} \int d^2q_{\perp} \frac{1}{\sigma_{el}} \frac{d\sigma_{el}^{\text{med}}}{d^2q_{\perp}} \\ \times \frac{2\mathbf{k}_{\perp} \cdot \mathbf{q}_{\perp}}{[k_{\perp}^2 + m^2][(k_{\perp} - q_{\perp})^2 + m^2]} \left[ 1 - \cos \frac{(\mathbf{k}_{\perp} - \mathbf{q}_{\perp})^2 + m^2}{xp_0^+} \Delta z \right]$$

The ambiguity is removed by the off-diagonal splittings. Bottom line: x m corrections in the poles and interference phases but dropped in numerator

F. Ringer et al . (2016)

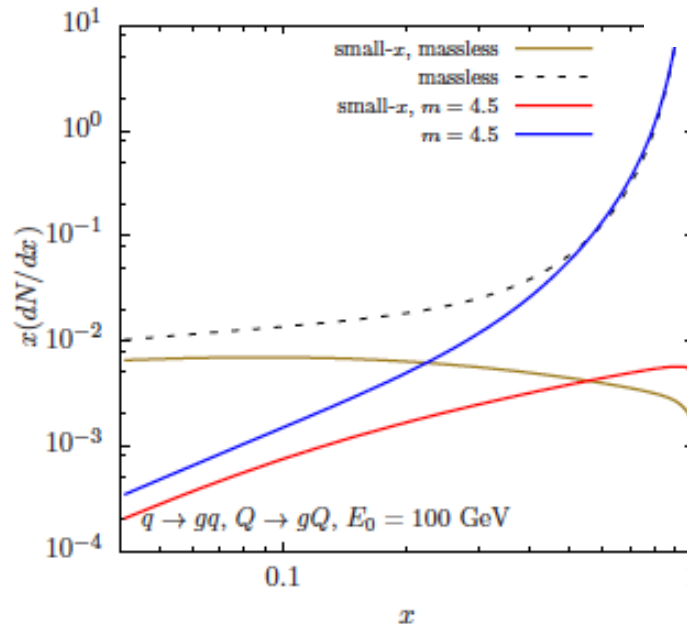
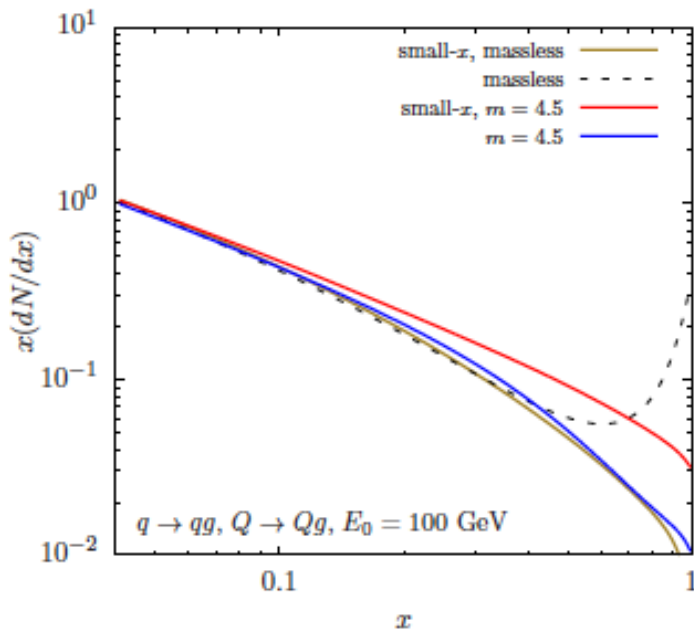
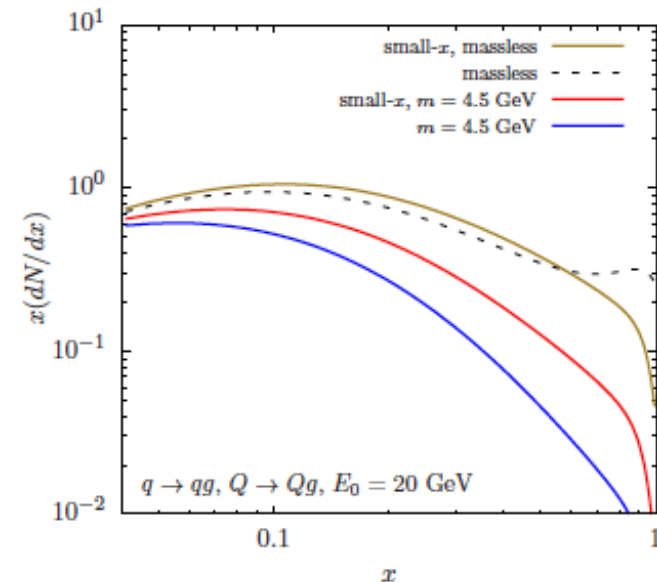
# Numerical comparison

For  $p_T$  10-20 GeV even at “small- $x$ ” there is a difference

The –off-diagonal splittings are not small

F. Ringer et al. (2016)

Cannot be interpreted as energy loss: NLO implementation, evolution



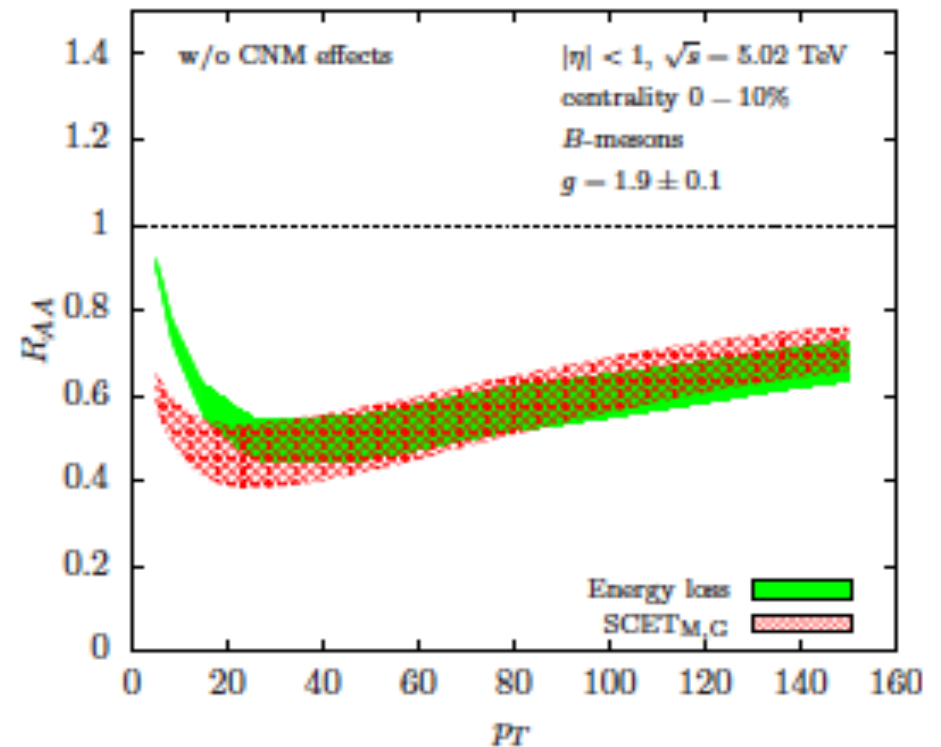
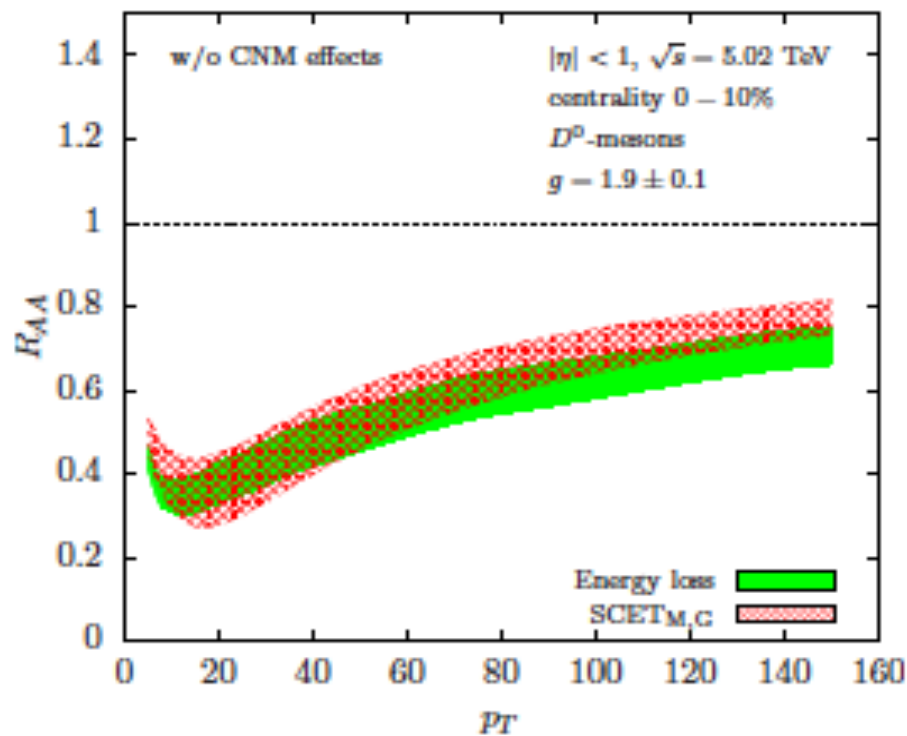
Qualitatively the same behavior of RAA / from light

Different  $g \sim 5\%$   
Different  $q_{hat} \sim 20\%$

Expect at least the same Uncertainty at least

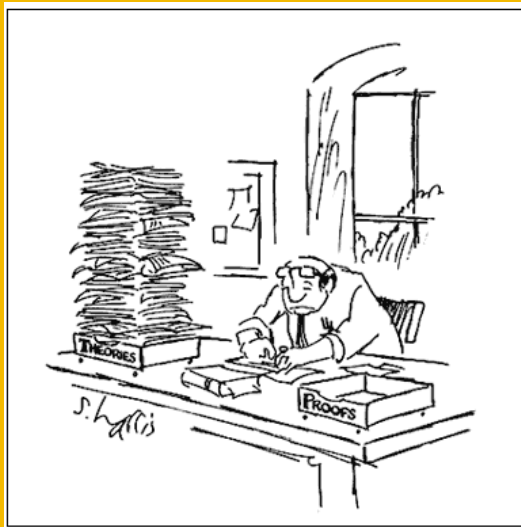
# Combined uncertainty

Includes both production mechanism and e-loss vs parton shower



At high  $p_T$  there is at least 20% combined uncertainty. Did not increase much since gluon fragmentation in H is softer and offsets the difference between quark-gluon energy loss. At low  $p_T$  the uncertainties can grow to 30% D and 50 + % B. Does it further affect collisional interactions?

# III. Differences between models



# Generally 4 energy loss approaches

- The scattering lengths and momentum transfers are largely independent, providing a 2D parameter space. Such scenario would require expensive multi-parameter fits to data and has not been explored so far in the literature.
- Assuming local thermal equilibrium, density and temperature can be related at any space time point. The range of the interaction and parton scattering cross section can be estimated and depend on the typical coupling between the jet and the medium  $g$ . The scattering length is then obtained from the QGP density and the scattering cross section
- One can relate in thermal field theory all relevant medium parameters to the temperature  $T$ . In spirit, this is similar to the situation described above but in this case the scattering cross sections and densities are not explicitly evaluated.



# Generally 4 energy loss approaches

- An approach to energy loss set in the limit of infinite energies and infinite number of scatterings assumes that at any scale the transverse momentum broadening of any scale is given purely by 2D Gaussian random. By discarding the detailed kinematic information that pertains to parton scattering one can relate the radiative intensity spectra to the transport parameter  $\hat{q}$
- In deep inelastic scattering the radiative spectrum can be related to higher twist matrix elements of field operators. The scattering length can be thought of as the inter-nucleon distance. The application to the QGP case is by analogy.

Improvements have been made in e-loss models. Papers have been written how model  $A=C$ ,  $B=D$ . When it comes to application there are differences

General agreement on the form of the mass correction on the diagonal energy loss piece

Dokshitzer et al. (2001)

Djordjevic et al. (2003)

# IV. The story of q-hat

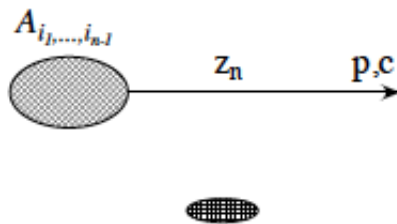


# Transverse momentum broadening of partons

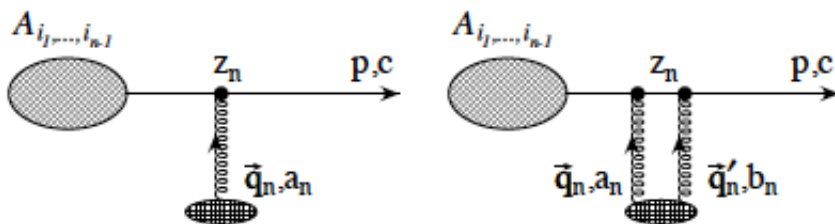
The transport parameter  $\hat{q}$  is discussed in the context of transverse momentum broadening.

Baier et al. (1997)

M. Gyulassy et al. (2002)



Calculate diagrams of this type  
Use the reaction operator approach to sum the interactions in impact parameter space



Note that these are equivalent representations

$$dN(\mathbf{p}) = e^{-\sigma_{el}T(\mathbf{b}_0)} \int d^2\mathbf{b} e^{i\mathbf{p}\cdot\mathbf{b}} e^{\bar{\sigma}_{el}(\mathbf{b})T(\mathbf{b}_0)} dN^{(0)}(\mathbf{b})$$

$$= \sum_{n=0}^{\infty} e^{-\chi} \frac{\chi^n}{n!} \int \prod_{i=1}^n d^2\mathbf{q}_i \frac{1}{\sigma_{el}} \frac{d\sigma_{el}}{d^2\mathbf{q}_i} dN^{(0)}(\mathbf{p} - \mathbf{q}_1 - \dots - \mathbf{q}_n)$$

# Pure random walk approximation

The approximation arises as follows

M. Gyulassy et al . (2002)

$$\frac{d\tilde{\sigma}_{el}}{d^2\mathbf{q}}(\mathbf{b}) = \int \frac{d^2\mathbf{q}}{(2\pi)^2} e^{-i\mathbf{q}\cdot\mathbf{b}} \frac{1}{\pi} \frac{\mu^2}{(\mathbf{q}^2 + \mu^2)^2} = \frac{\mu b}{4\pi^2} K_1(\mu b) \approx \frac{1}{4\pi^2} \left( 1 - \frac{\xi \mu^2 b^2}{2} + \mathcal{O}(b^3) \right)$$

$$\xi = 2 \ln(1.08 / \mu b)$$

Not only are sub-leading powers of energy neglected but also the leading logarithmic dependence must be neglected.

One must neglect the energy dependence

$$dN(\mathbf{p}) = \int d^2\mathbf{b} e^{i\mathbf{p}\cdot\mathbf{b}} \frac{1}{(2\pi)^2} \frac{e^{-\frac{\chi \mu_D^2 \xi b^2}{2}}}{\chi \mu_D^2 \xi} = \frac{1}{2\pi} \frac{e^{-\frac{p^2}{2\chi \mu_D^2 \xi}}}{\chi \mu_D^2 \xi}$$

$$\langle \mathbf{p}^2 \rangle = 2\chi \mu_D^2 \xi, \quad \xi \sim O(1)$$

The transport parameter  $\hat{q}$

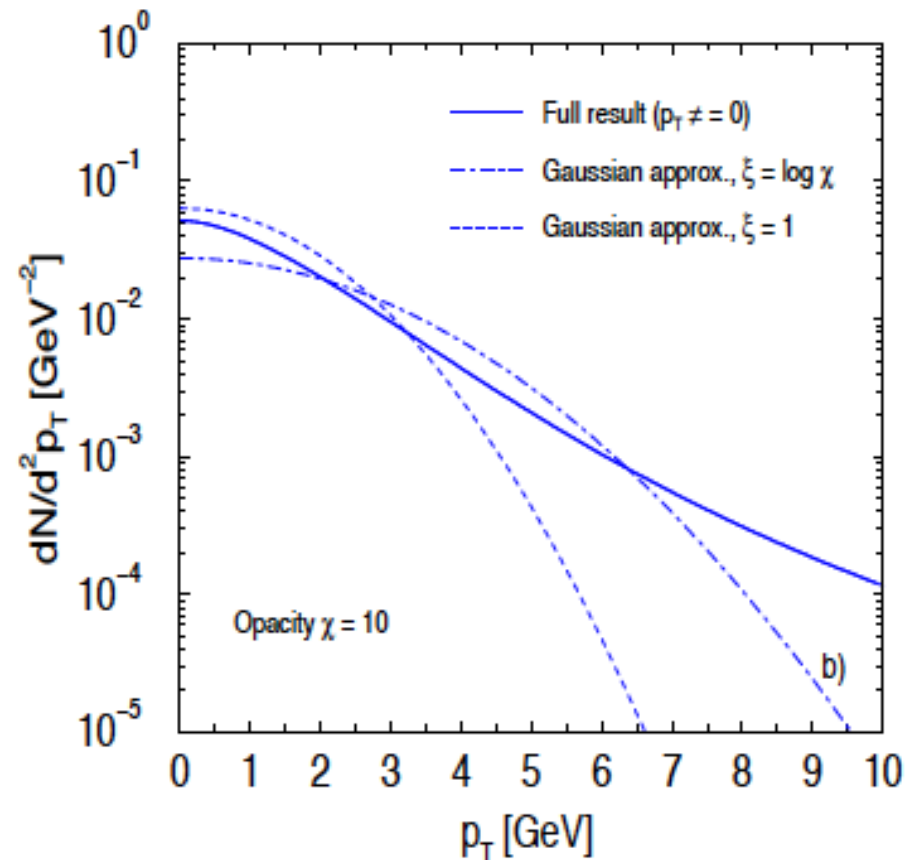
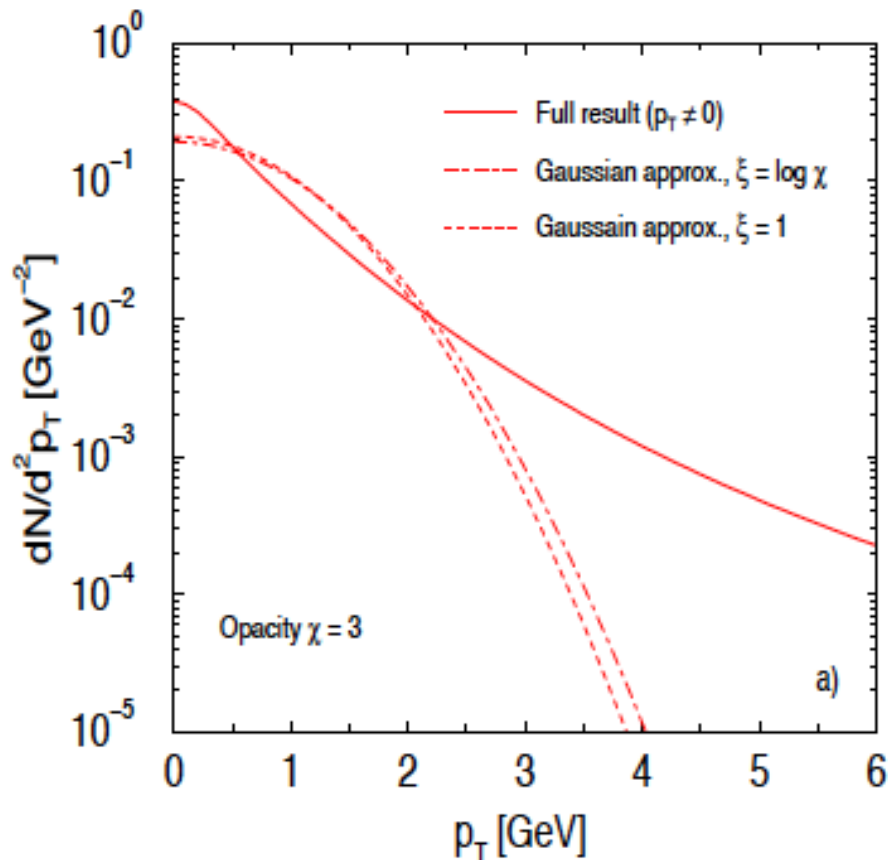
$$\langle \mathbf{p}^2 \rangle = \int \hat{q}(z) d\Delta z, \quad \hat{q}(z) = 2 \frac{\mu_D^2(z)}{\lambda_g(z)}$$

Only in this original definition  $\hat{q}$  is related to the (transport) properties of the medium

# Does it really describe parton broadening

No it doesn't

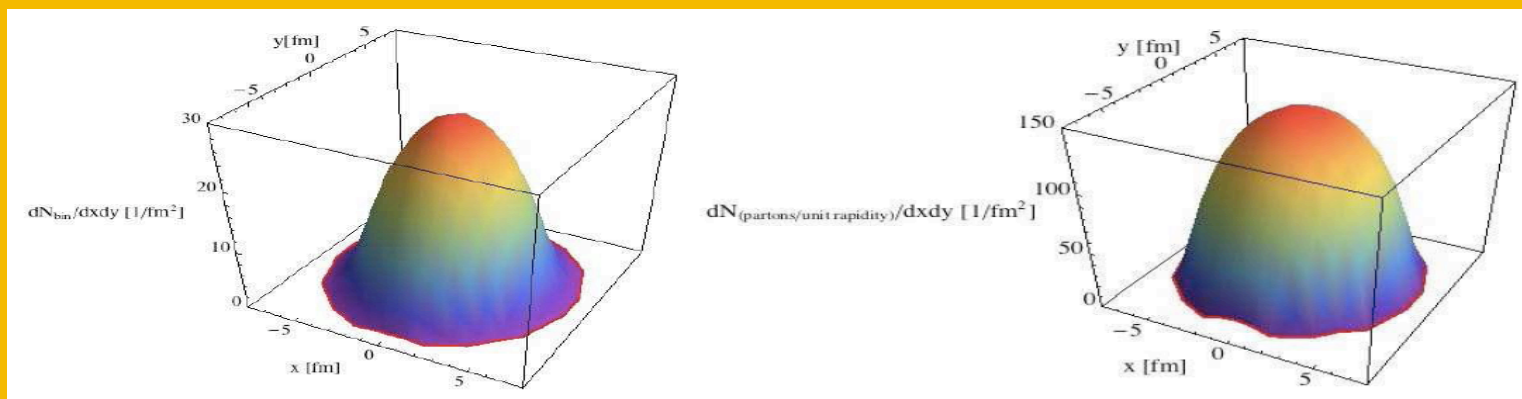
M. Gyulassy et al. (2002)



Certainly does not capture the tails of the distributions. But also does not work all that great for realistic opacities even at small  $p_T$

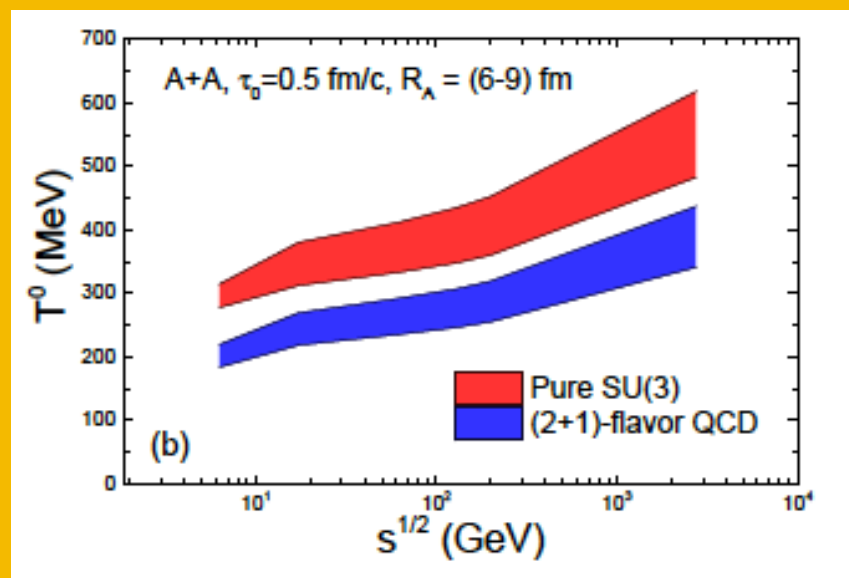
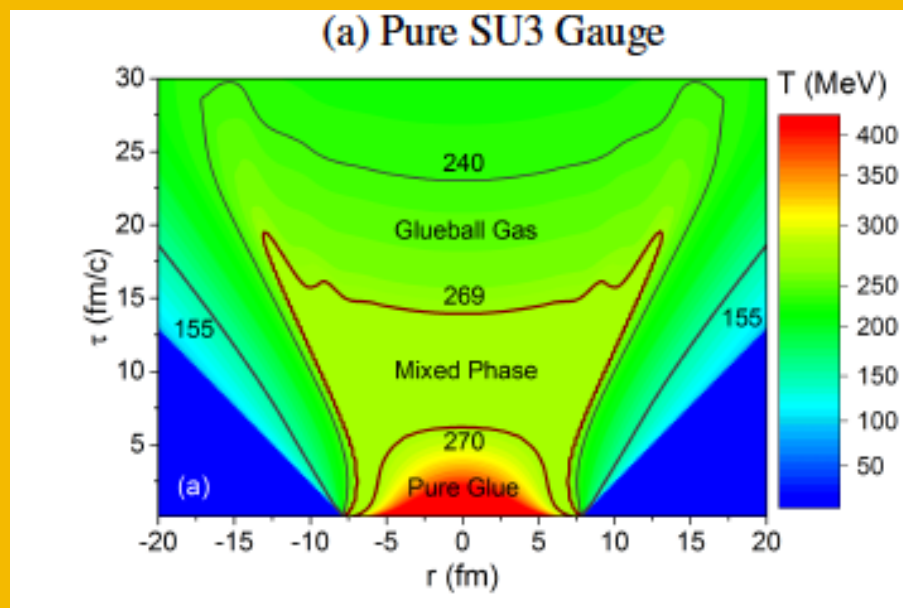
# Uncertainties in quoting $\hat{q}$

- Most theoretical energy loss models do not have  $\hat{q}$  as input. The prescription of how to wither implement it or quote it is a systematic uncertainty not possible to quantify, at least 100%
- Energy dependence should be removed by either quoting the Gaussian part or using working prescriptions how to cancel the leading logarithmic energy dependence. (some models do include energy dependence, this is not useful as characterization of the medium)
- $\hat{Q}$ -hat (or any other parameter,  $T$ , Debye screening, scattering length, energy density) depends on the space-time point.



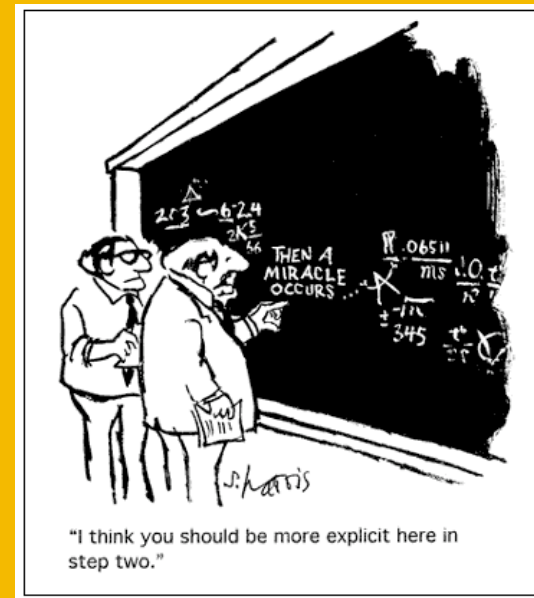
# Uncertainties in quoting $\hat{q}$

- If an average is quoted it should be specified, how are the different space-time points weighted
- Also may depend on the tupe of hydrodynamic medium, gluon dominated, quark-gluon, how many degrees of freedom



V. Vovchenko et al. (2002)

# V. Non-locality of in-medium parton splittings / radiative energy loss





# Conclusions

- Uncertainties in heavy flavor tomography. Production mechanism of heavy flavor, energy loss vs parton showers. At high  $p_T$  these are quantifiable uncertainties.  $\sim 20\text{-}30\%$  uncertainty in the extraction of  $q$ -hat. At smaller  $p_T$  differences can be  $50\text{-}100\%$  because of gluon fragmentation
- Uncertainties due to theoretical models. They take different approximations. When compared at face value they were  $400\%$  different. Even of improvements made –  $100\%$  difference
- Most models don't use  $q$ -hat. Different prescriptions are used for different models. This is hard to quantify.  $100\%$  uncertainty. The values of  $q$ -hat depend of the space-time point or average, what average, etc. Are logs of energy eliminated or not.  $100\%$
- Within a specific model realistically one can quote transport parameters with  $20\%$  accuracy (if everything is specified)
- Between models I don see how better than  $100\%$  uncertainty can be achieved. Anything smaller would appear to be "optimistic"

# 2017 Jets and heavy flavor workshop

- Second in a series of workshops to bring the NP and HEP communities working on jets and heavy flavor, with emphasis on QCD and SCET

## Santa Fe Jets and Heavy Flavor Workshop

February 13-15, 2017

### Workshop topics:

- Jets and jet substructure in hadronic and nuclear collisions
- Heavy flavor production in p+p, p+A and A+A
- Perturbative QCD and SCET
- New theoretical developments
- Recent experimental results from RHIC and LHC

Contact: [sfjet17@lanl.gov](mailto:sfjet17@lanl.gov)

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