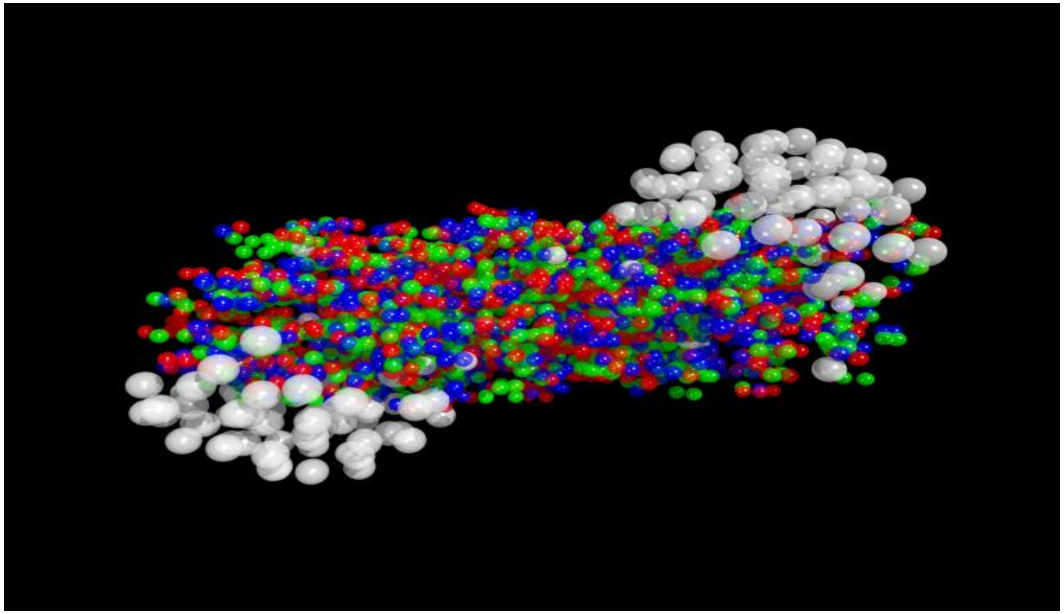
University of Catania
INFN-LNS, Catania, Italy

EMMI-RRTF:Heavy quark dynamics



Santosh K. Das

In collaboration with: Vincenzo Greco
Francesco Scadina
Salvatore Plumari

Outline of my talk..............

- Model description
- Bulk observables
- Langevin vs Boltzmann with M/T
- Hadronization

Heavy quark initialization and momentum evolution:

✧r-space: Ncoll (Glauber mode)

✧p-space: FONLL

Langevin dynamics

$$dx_j = \frac{p_j}{E} dt$$

$$dp_j = -Ap_j dt + \sqrt{dt} C_{jk}(t, p + \xi dp) \rho_k$$

A **is the deterministic friction (drag) force.**

C_{ij} **is stochastic force .**

Heavy quark-bulk interactions :
We use the pQCD transport coefficients provided by the organizer.

We use $\xi = 1$, **the post-point Ito interpretation of momentum argument.**

The momentum updates in the Langevin equation have heen calculated in the local fluid rest frame.

It is necessary to know the radial flow and temperature associated with the position (x,y,z,t) of the HQ in the given position. In our approach this is provided by the solution of the Boltzmann equation for the bulk

$$(x,y,z,t)_{HQ} \rightarrow (\beta_x, \beta_y, \beta_z, T, \rho, \varepsilon)$$

Motivation for Transport approach

$$\left\{p^{*\mu}\partial_{\mu}+\left[p^{*}_{\nu}F^{\mu\nu}+m^{*}\partial^{\mu}m^{*}\right]\partial^{p^{*}}_{\mu}\right\}f(x,p^{*})=C[f]$$

Free streaming Field Interaction Collisions -> $\eta \neq 0$

- **S**tarting from 1-body distribution function f(x,p) and not from $T_{\mu\nu}$:
  - **f(x,p) out-of-equilibrium: CGC-Qs scale or high p_T**
  - **Extract viscous correction δf to f(x,p)**
  - **Relevant at LHC due to large amount of minijet production**
  - **Freeze-out self-consistently related to $\eta/s(T)$**
  - **HQ dynamics in the same framework**

DISADVANTAGES?!

- **Relaxation times fixed by kinetic theory**
- **Hadronization needed: coal.+frag . under progress**

Simulate at fixed shear viscosity

Usually input of a transport approach are *cross-sections and fields*, but here we reverse it and start from η/s with aim of creating a more direct link to viscous hydrodynamics

Chapmann-Enskog

$$\frac{\eta}{s} = \frac{1}{15}\langle p\rangle \cdot \tau_\eta = \frac{1}{15}\frac{\langle p\rangle}{g(\frac{m_D}{T})\sigma_{TOT}\rho}$$

$$g(a) = \frac{1}{50}\int dy y^6 \left[(y^2+\frac{1}{3})K_3(2y) - yK_2(2y)\right] h\left(\frac{a^2}{y^2}\right)$$

$g(a=m_D/2T)$ correct function that fix the relaxation time for the shear motion

Transport code

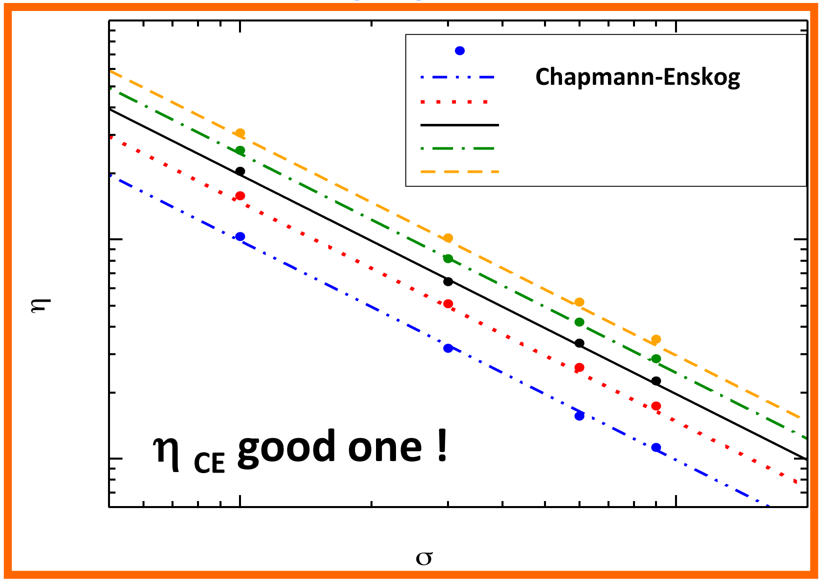
$$\sigma_{tot}(n(\vec{r}),T) = \frac{1}{15}\frac{\langle p_\alpha\rangle}{g(a)n_\alpha}\frac{1}{\eta/s}$$

Space-Time dependent cross section evaluated locally

$0 < g(m_D/2T) < 2/3$

forward peaked — Isotropic $m_D \to \infty$

Chapman-Enskog agrees with Green-Kubo



S.Plumari et al., PRC86(2012

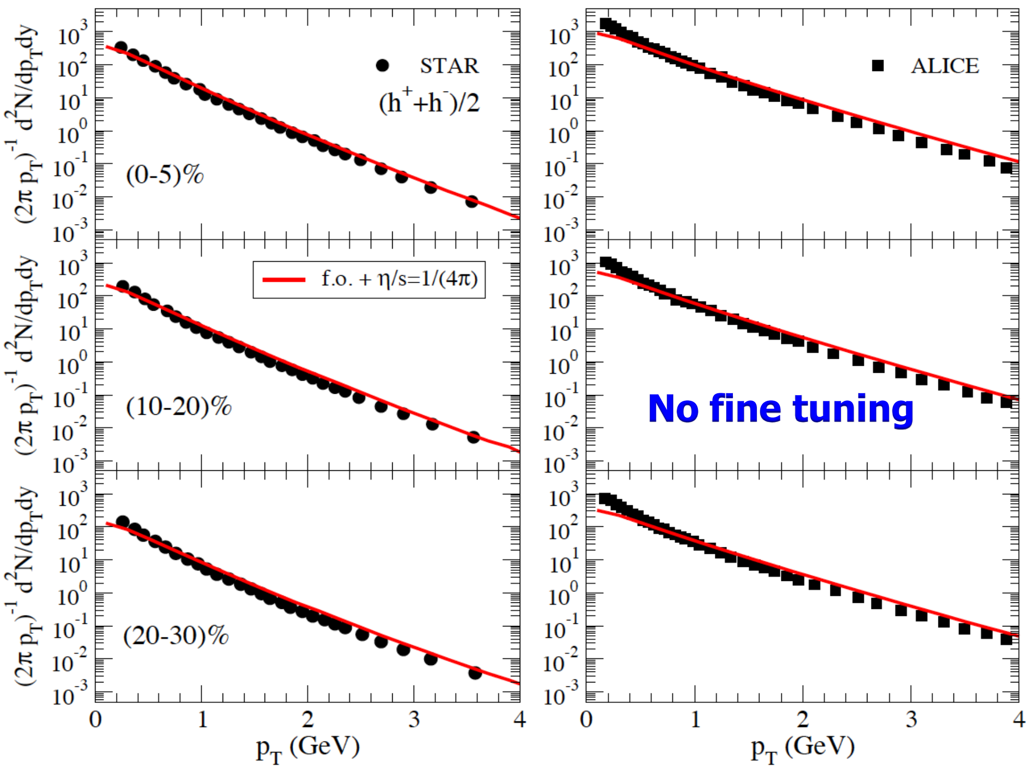
Bulk Initial Conditions

✧ r-space: standard Glauber model

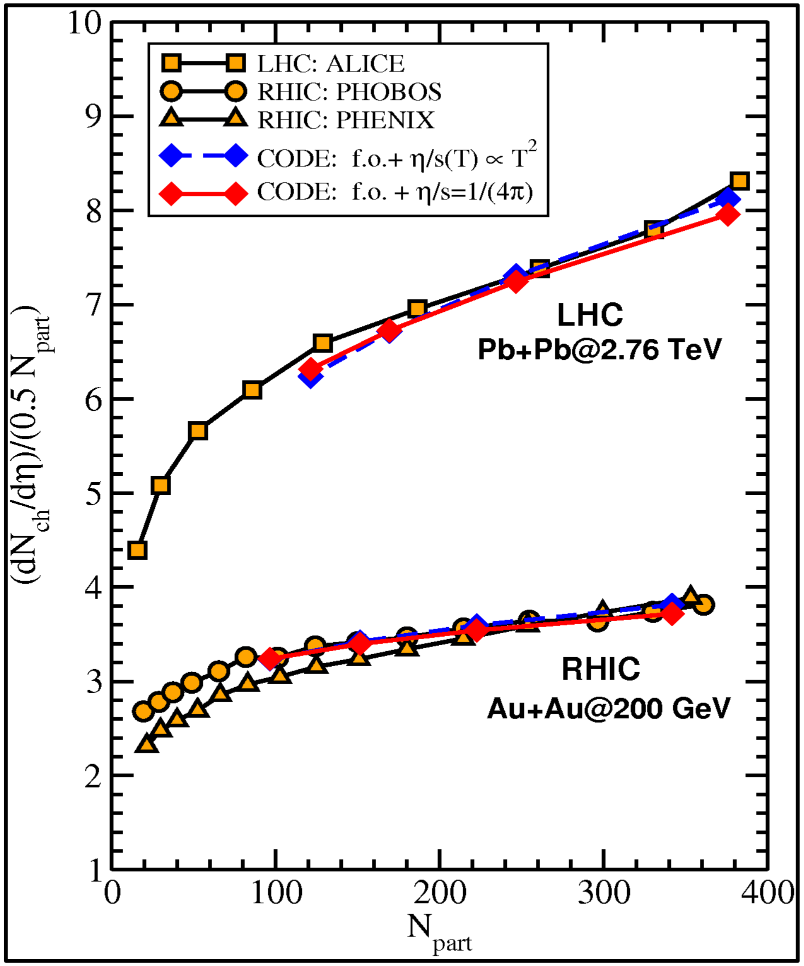
✧ p-space: Boltzmann-Juttner at T+ minijet [p_T>3GeV]

Typical hydro condition

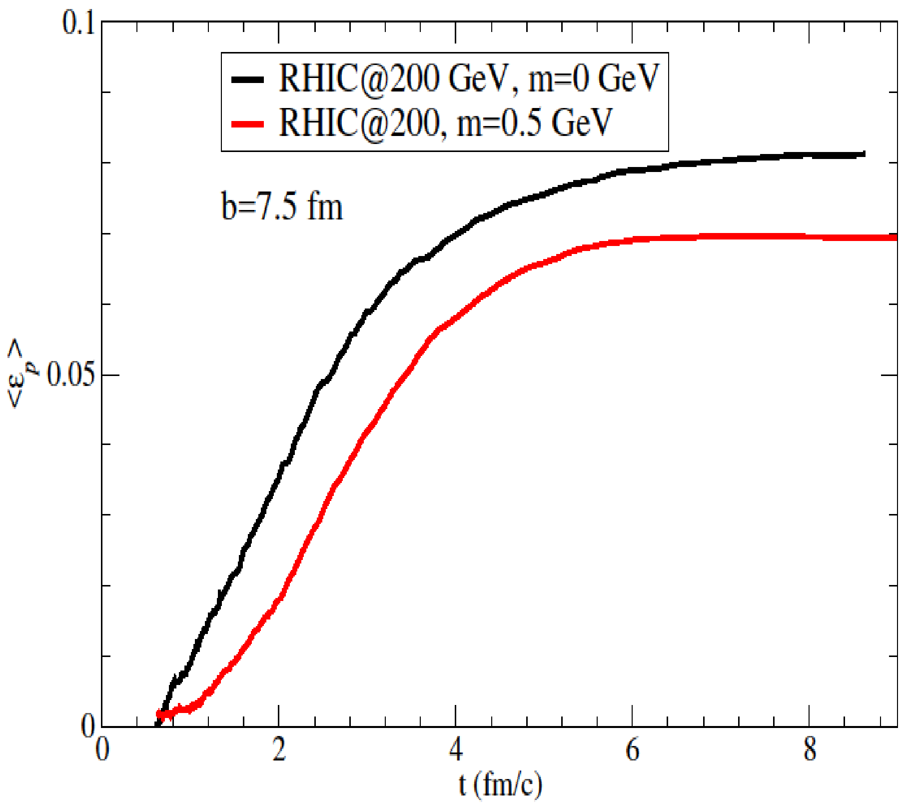
	62 GeV	200 GeV	2.76 TeV
T_0	290 MeV	340 MeV	510 MeV
τ_0	0.7 fm/c	0.6 fm/c	0.3 fm/c

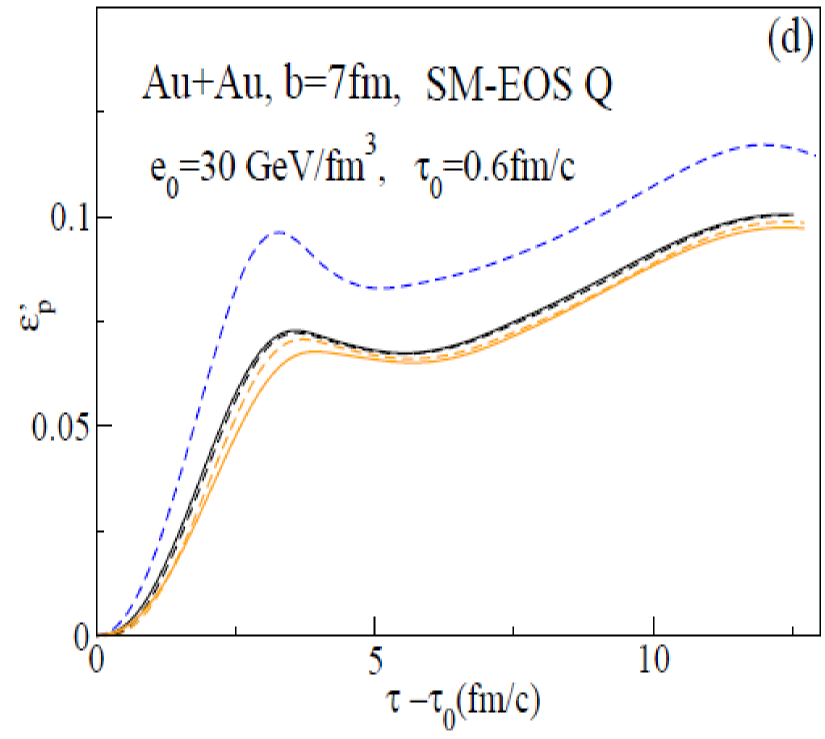


Spectra and multiplicity



Eccentricity evolution

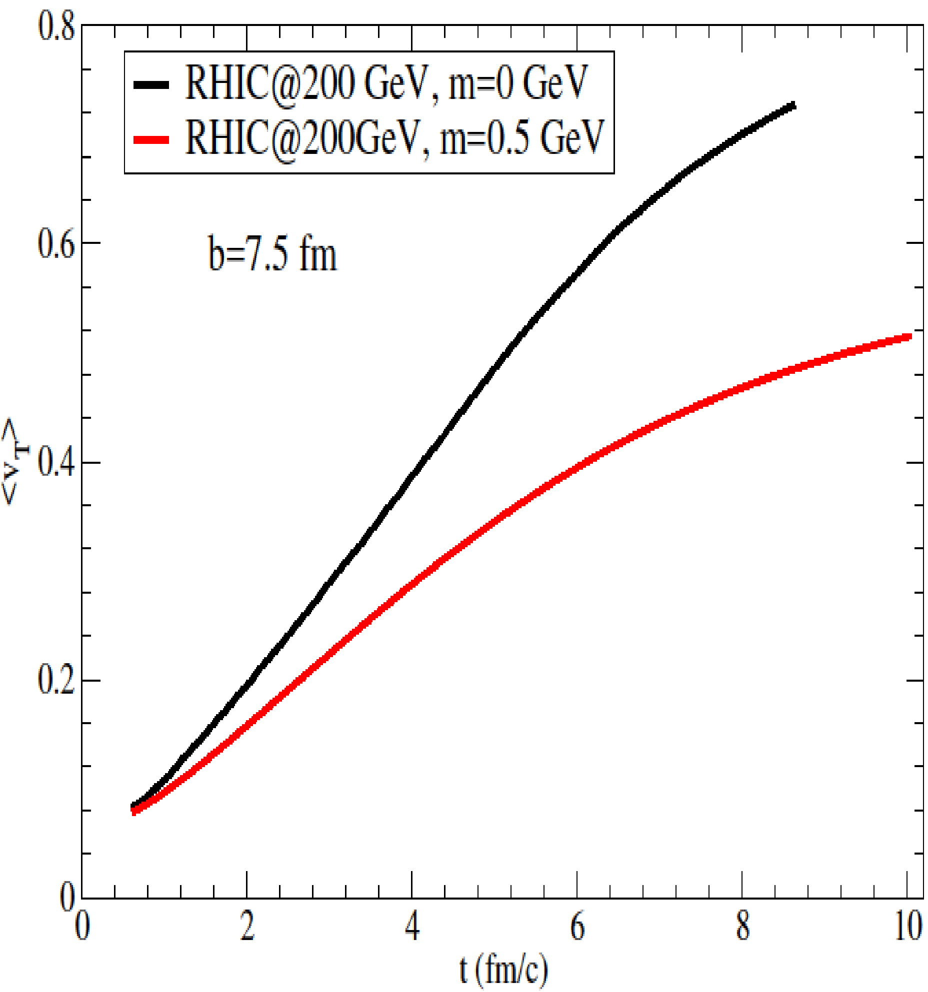


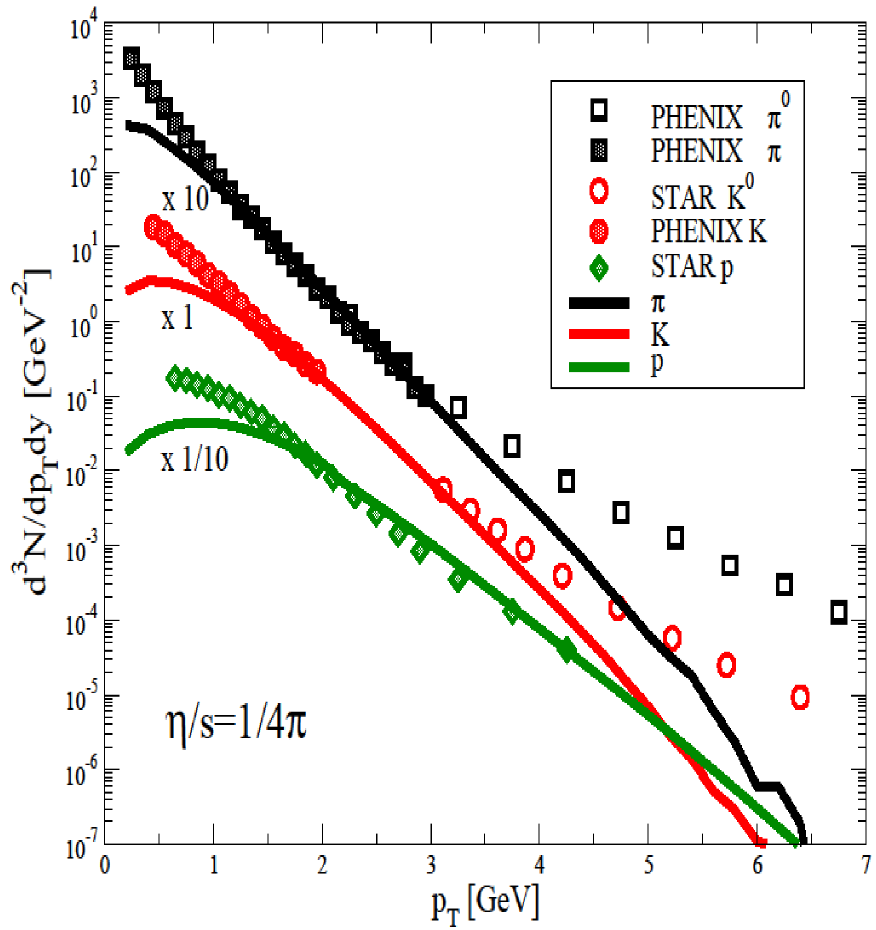


Song and Heinz
PRC,78, 024902, 2008

The eccentricity is around 20% small in viscous hydro than ideal.

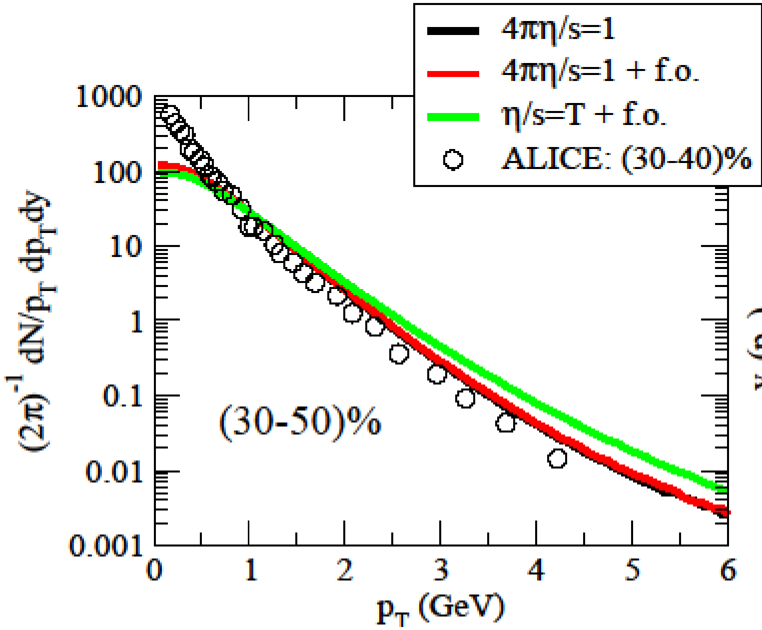
Radial flow and hadronization with Coalescence

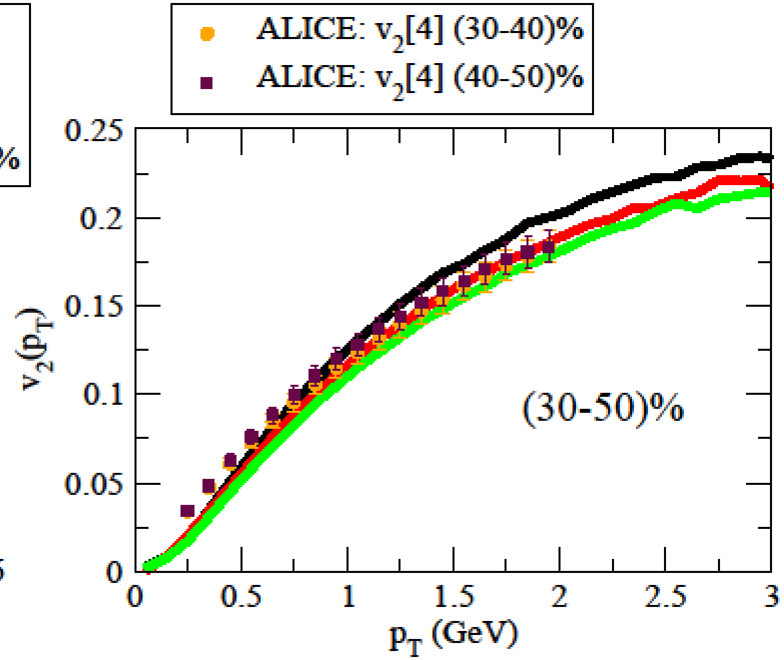


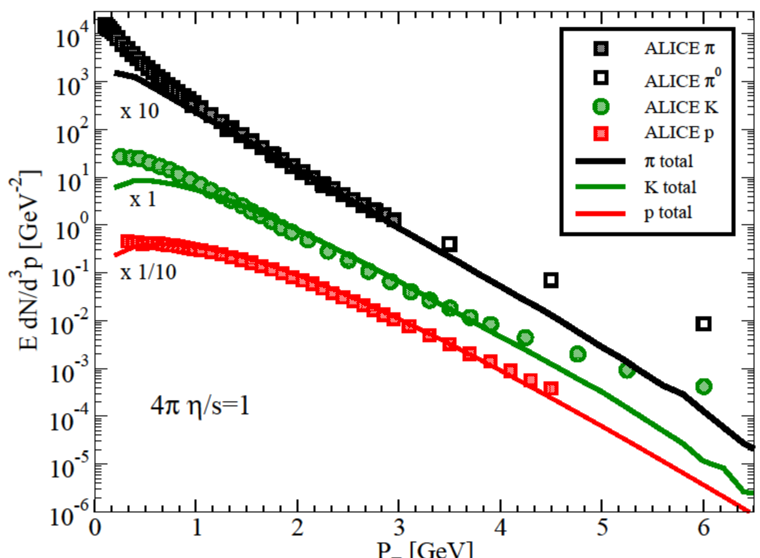


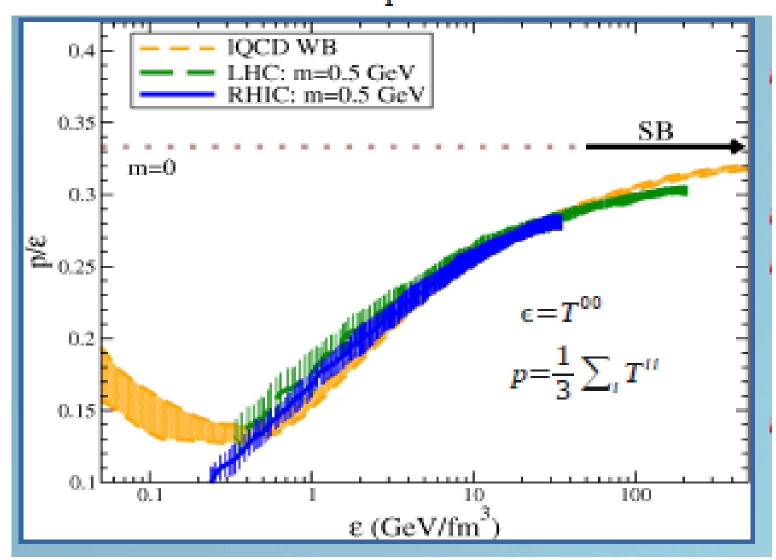
Fragmentation and resonances are missing.

Spectra and elliptic flow (RRTF assignments)



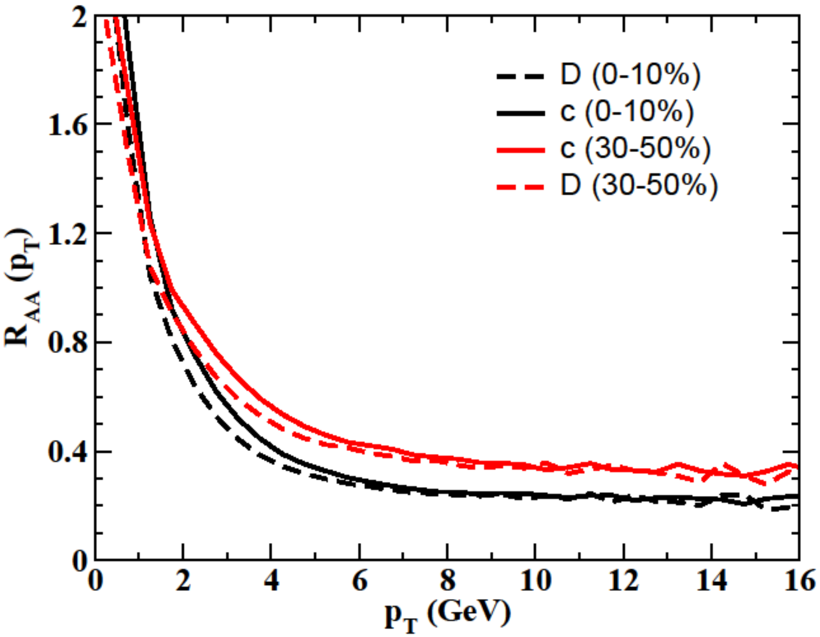


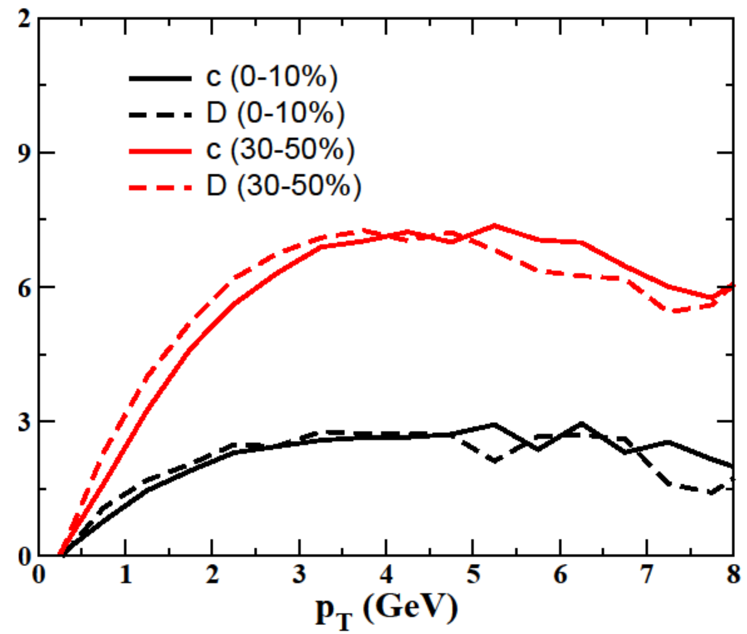




Heavy quark RAA and v2
(RRTF assignments)

At Tc, Peterson function has been used for heavy quark fragmentation:





In central collisions we are close to hydro results.
In peripheral collision our collision rate decrease, hence, our mean free path increase.
This implies a larger average T in peripheral collisions than hydro.

Langevin and Boltzmann approaches to heavy quark:

Boltzmann Kinetic equation

$$\left(\frac{\partial}{\partial t}+\frac{P}{E}\frac{\partial}{\partial x}+\mathrm{F}.\frac{\partial}{\partial p}\right)f\left(x,p,t\right)=\left(\frac{\partial f}{\partial t}\right)_{col}$$

- **The plasma is uniform ,i.e., the distribution function is independent of x.**
- **In the absence of any external force, F=0**

$$R(p,t)=\left(\frac{\partial f}{\partial t}\right)_{col}=\int d^3k\left[\omega(p+k,k)f(p+k)-\omega(p,k)f(p)\right]$$

$$\omega(p,k)=g\int\frac{d^3q}{(2\pi)^3}f'(q)v_{q,p}\sigma_{p,q\to p-k,q+k}$$

→ is rate of collisions which change the momentum of the charmed quark from p to p-k

$$\omega(p+k,k)f(p+k)\approx\omega(p,k)f(p)+k.\frac{\partial}{\partial p}(\omega f)+\frac{1}{2}k_ik_j\frac{\partial^2}{\partial p_i\partial p_j}(\omega f)$$

$$\frac{\partial \mathbf{f}}{\partial \mathbf{t}}=\frac{\partial}{\partial \mathbf{p_i}}\left[\mathbf{A_i(p)f}+\frac{\partial}{\partial \mathbf{p_j}}\left[\mathbf{B_{ij}(p)f}\right]\right]$$

B. Svetitsky PRD 37(1987)2484

where we have defined the kernels

,

$$\mathbf{A_i}=\int \mathbf{d^3k\,\omega(p,k)k_i}\quad\rightarrow \textbf{Drag Coefficient}$$

$$\mathbf{B_{ij}}=\int \mathbf{d^3k\,\omega(p,k)k_ik_j}\rightarrow \textbf{Diffusion Coefficient}$$

Heavy quark momentum evolution: Langevin vs Boltzmann

$$\omega(p+k,k)f(p+k) \approx \omega(p,k)f(p) + k.\frac{\partial}{\partial p}(\omega f) + \frac{1}{2}k_i k_j \frac{\partial^2}{\partial p_i \partial p_j}(\omega f)$$

Boltzmann Equation **Fokker Planck**

It will be interesting to study both the equation in a identical environment to ensure the validity of this assumption at different momentum transfer and their subsequent effects on RAA and v2.

Langevin dynamics:

$$dx_j = \frac{p_j}{E}dt$$

$$dp_j = -Ap_j dt + \sqrt{dt}C_{jk}(t, p+\xi dp)\rho_k$$

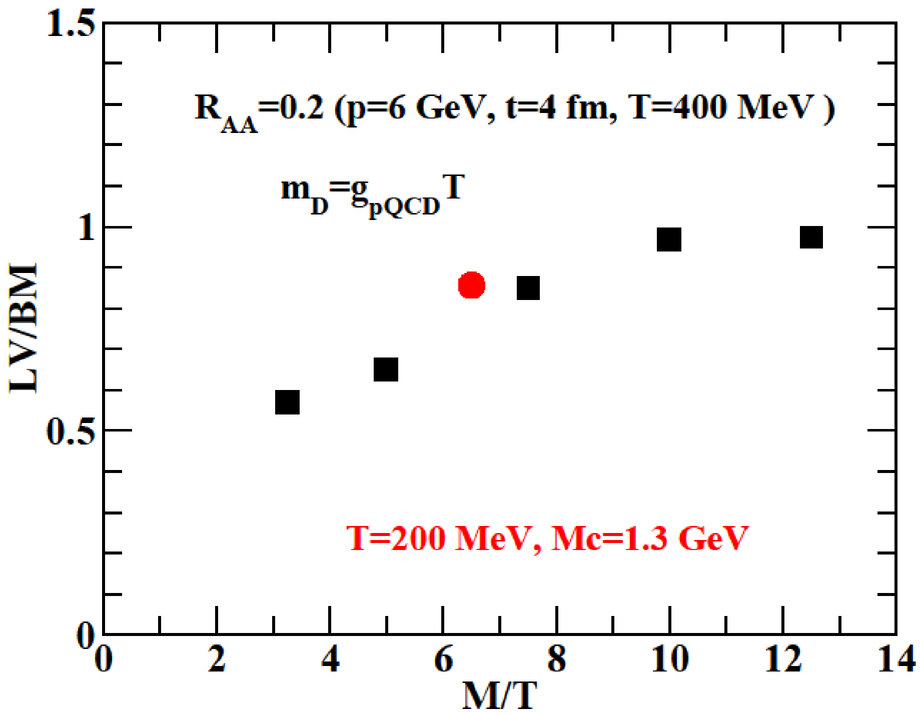
H. v. Hees and R. Rapp
arXiv:0903.1096

Γ **is the deterministic friction (drag) force**

C_{ij} **is stochastic force in terms of independent Gaussian-normal distributed random variable.**

Langevin vs Boltzmann at different M/T

Ratio between Langevin (LV) to Boltzmann spectra (BM):



We fixed the RAA within LV

Gluonic plasma, m_g=0

T=400 MeV, T=200 MeV

Mc=1.3, 2, 3, 4, 5 GeV

We use same initial charm distribution
For all the masses

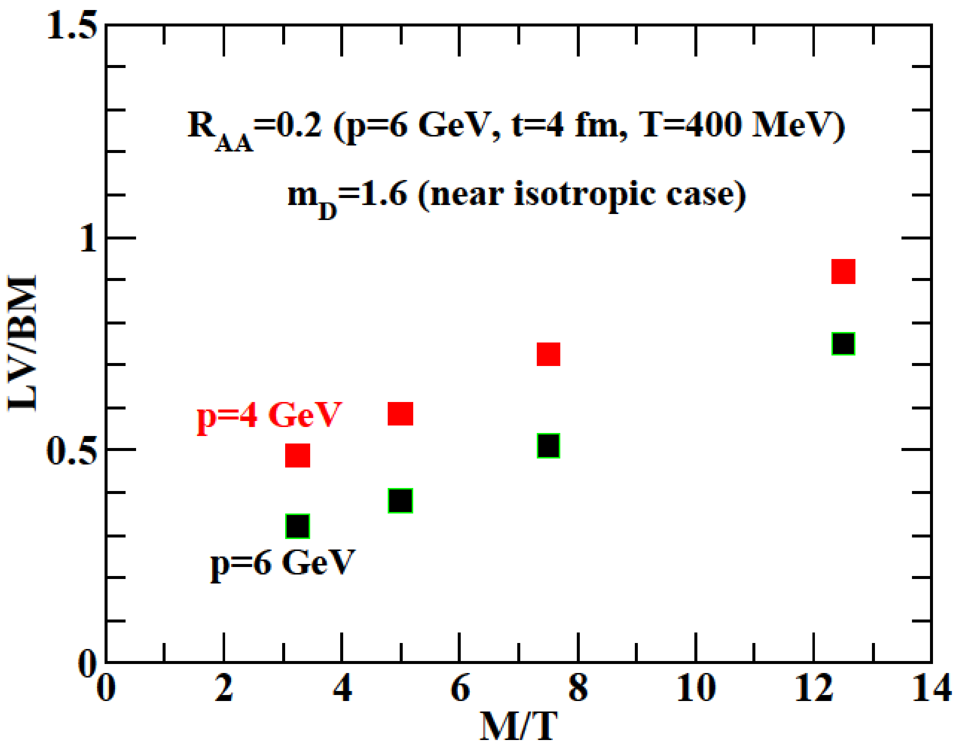
We change both the Mc as well as
the interaction.

We have plotted the results as a ratio between Langevin to Boltzmann to quantify how much the ratio deviates from 1.

Das, Scadrina, Plumari, Greco, PRC, 90, 044901 (2014)

Langevin vs Boltzmann at different M/T

Ratio between Langevin (LV) to Boltzmann spectra (BM):



We fixed the RAA within LV

Gluonic plasma, m_g=0

Mc=1.3, 2, 3, 5 GeV

We change both the Mc as well as the interaction.

Hadronization: Coalescence

$$\frac{d^2N_M}{d\mathbf{p}_T^2} = g_M \int \prod_{i=1}^{2} \frac{p_i \cdot d\sigma_i \, d^3\mathbf{p}_i}{(2\pi)^3 E_i} f_q(x_1, p_1) f_{\bar{q}}(x_2, p_2) \times f_M(x_1, p_1; x_2, p_2) \delta^{(2)}(\mathbf{p}_T - \mathbf{p}_{1T} - \mathbf{p}_{2T})$$

f_q invariant parton distribution function

- thermal with radial flow from transport

Hadron Wigner (Wave) function

$$f_M = 8\exp\left(-\frac{x^2}{\sigma^2}\right)\exp(-q^2\sigma^2) \quad (3)$$

First higher states (resonances) suppressed by the statistical exp[-(Δ

$x=x_1-x_2$ and $q^2=-(p_1-p_2)^2$

For D meson $\sigma_x = 1/\sigma_p$ fixed to $\langle r^2_D \rangle^{1/2} = 0.65$ fm

Greco, Rapp, Ko – PLB(2004)

$m_q=0.3$ GeV, $m_s=0.450$ GeV, mc=1.3 GeV

+ Fragmentation

For heavy quark fragmentation, we are using Peterson fragmentation :

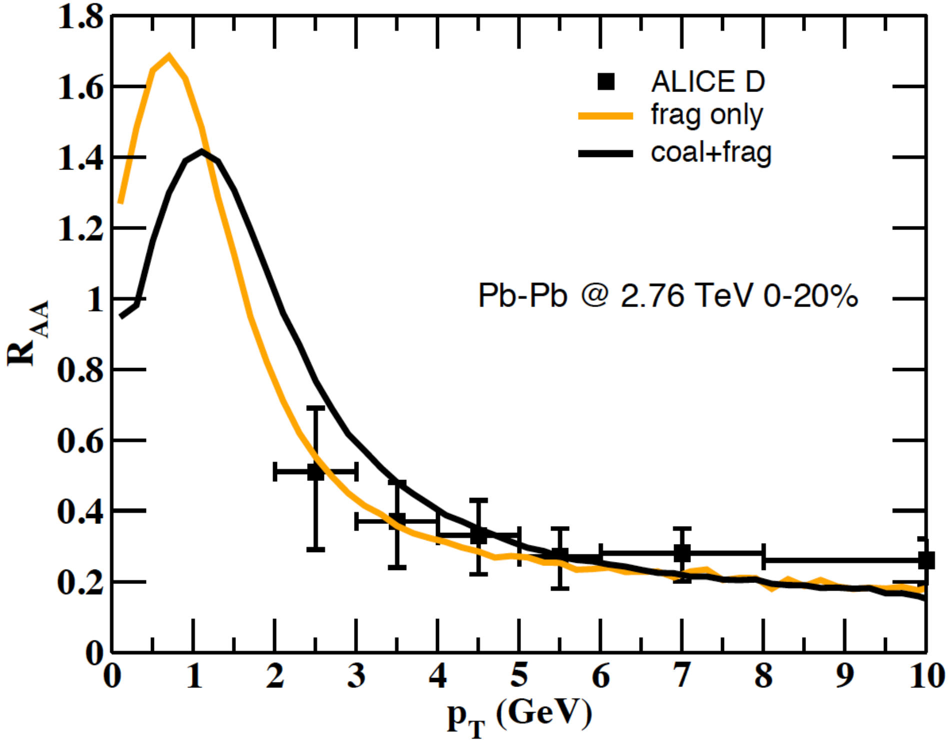
$$f(z) \propto \frac{1}{[z[1 - \frac{1}{z} - \frac{\epsilon_c}{1-z}]^2]} \quad (6)$$

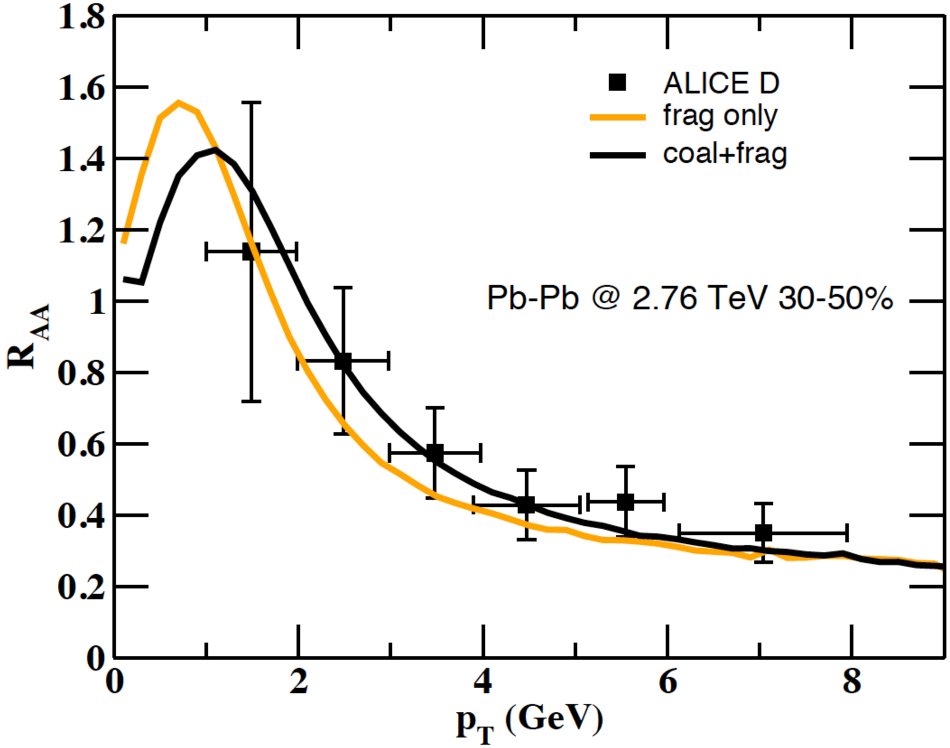
for charm quark $\epsilon_c = 0.04$. For bottom quark $\epsilon_c = 0.005$.

$P_{frag}(p_T) = 1 - P_{coal}(p_T)$

ALICE – D meson

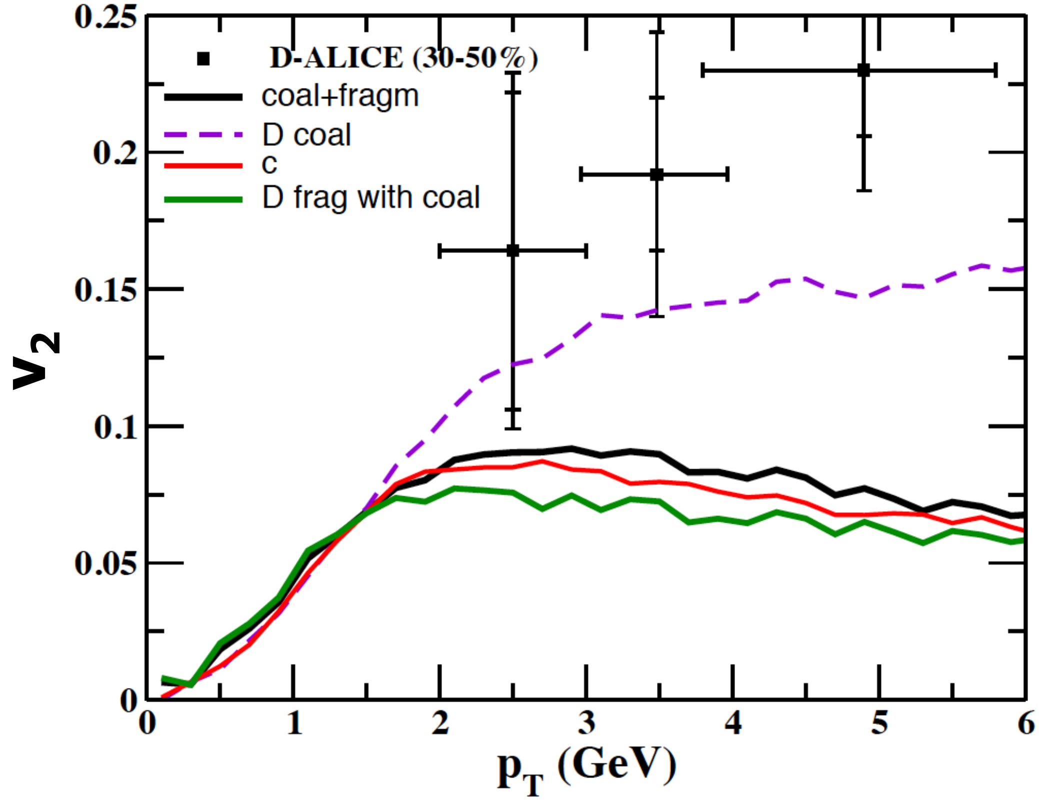
QPM - Boltzmann





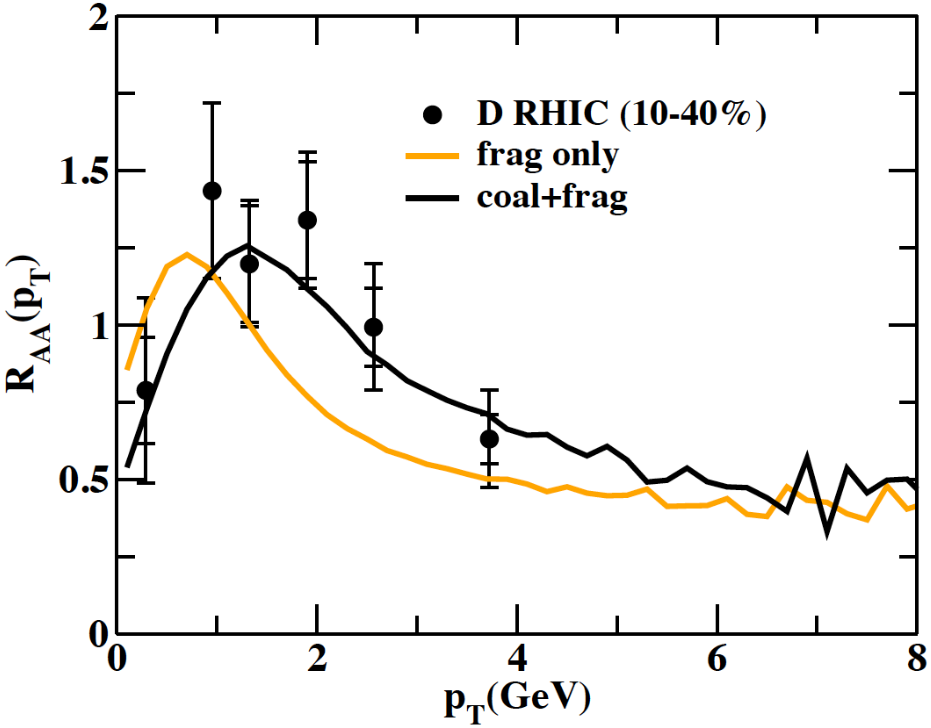
- ✧ No Hadronic Rescattering included
- ✧ Bump can be present also w/o coalescence
- ✧ Coalescence shift the bump

ALICE@2.76 ATeV

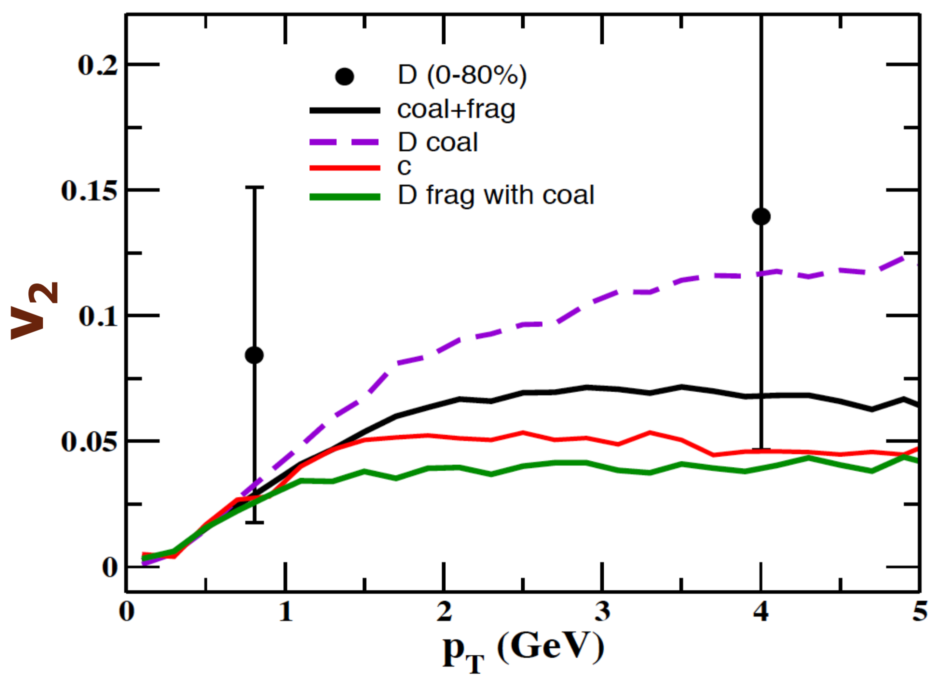


RHIC – D meson

QPM - Boltzmann

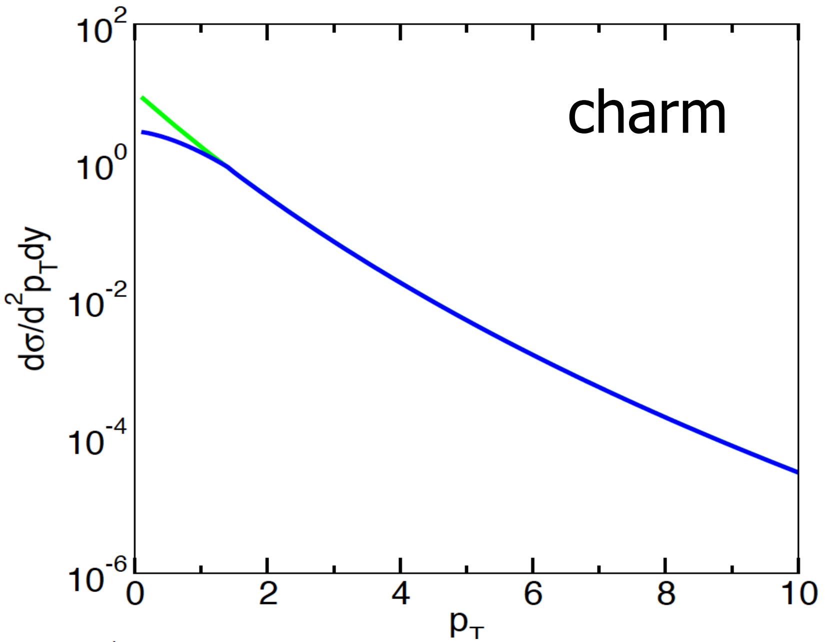


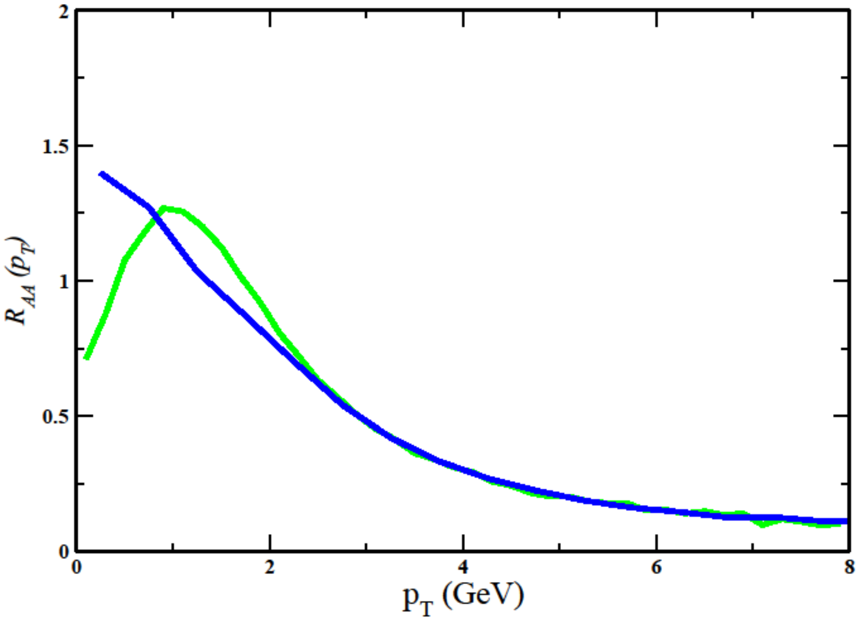
Au+Au @ 200 AGeV **0-80%**

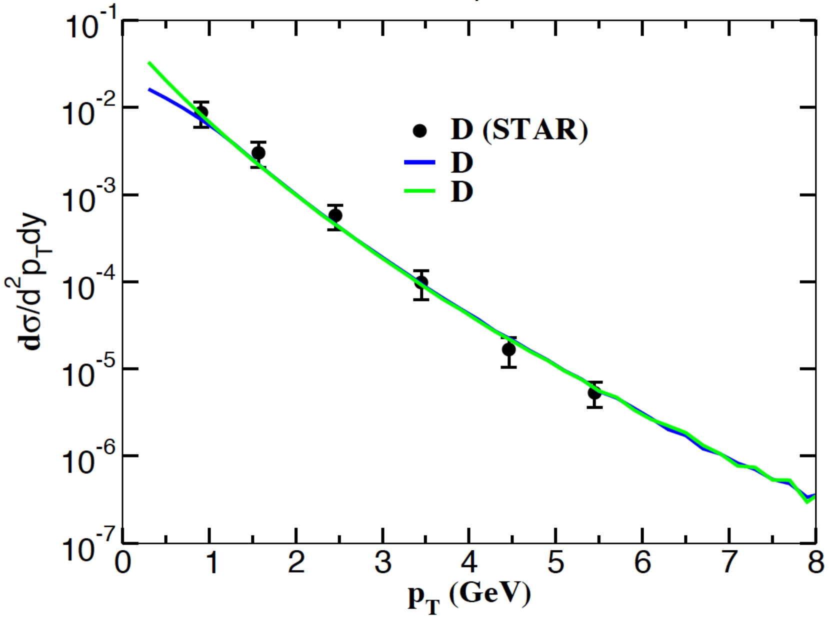


- ✧ No Hadronic Rescattering included
- ✧ Bump can be present also w/o coalescence
- ✧ Coalescence shift the bump

Impact of 2 dN/dp$_T$ well within FONLL

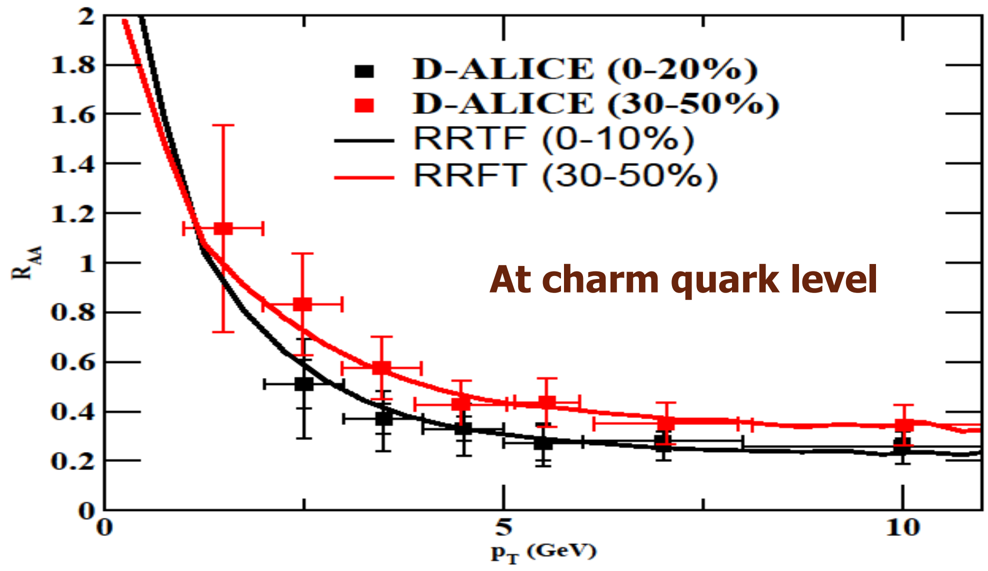


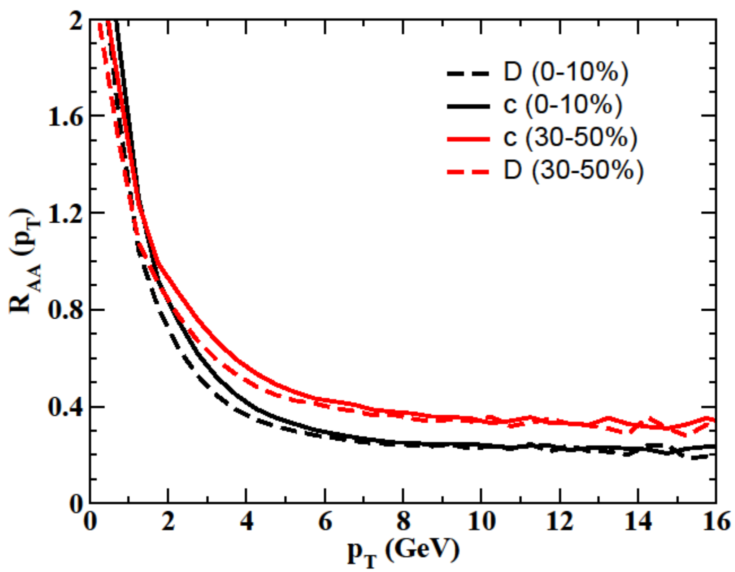


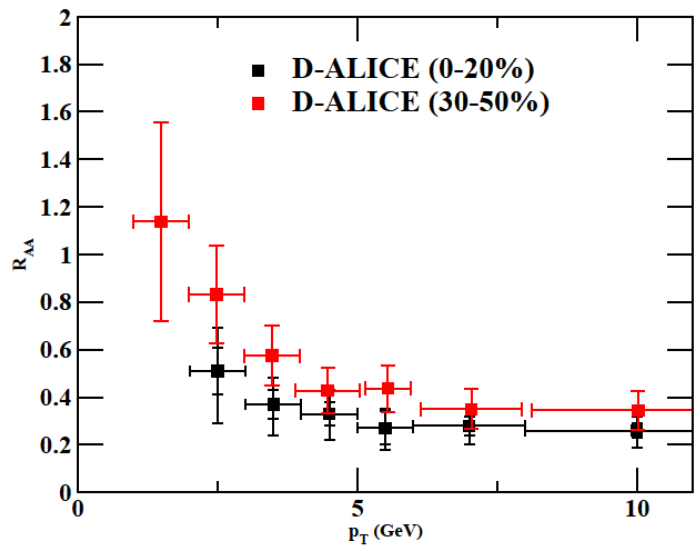


Especially for the bump
Look at distribution and data down to 0 P$_T$ essential


Thank You . . . .


D-ALICE (0-20%)
D-ALICE (30-50%)
RRTF (0-10%)
RRFT (30-50%)
At charm quark level
R_{AA}
p_T (GeV)

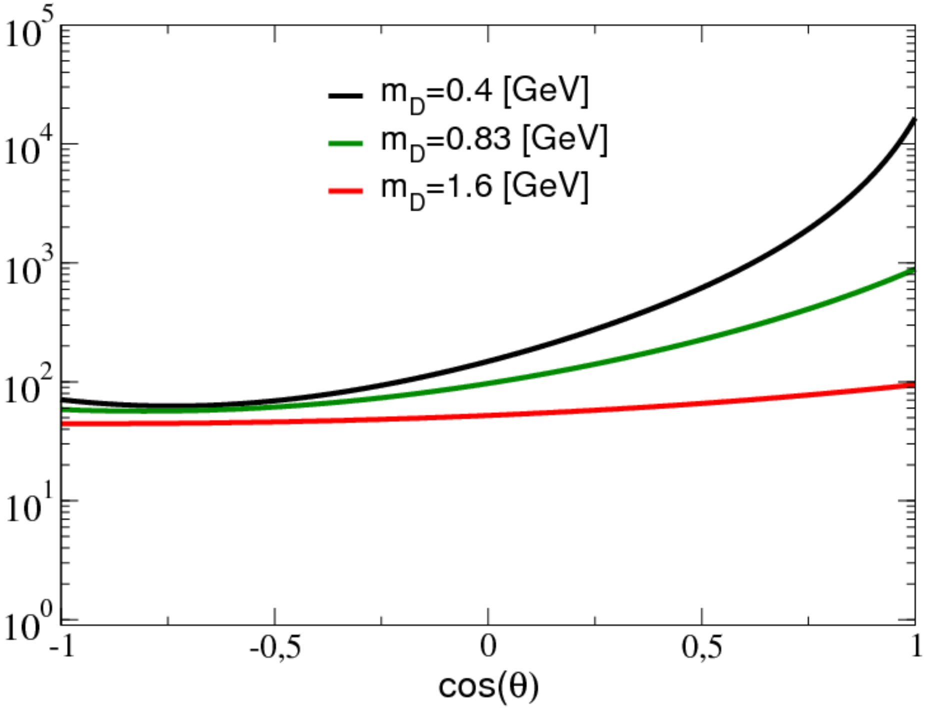

D (0-10%)
c (0-10%)
c (30-50%)
D (30-50%)
R_{AA} (p_T)
p_T (GeV)


D-ALICE (0-20%)
D-ALICE (30-50%)
R_{AA}
p_T (GeV)

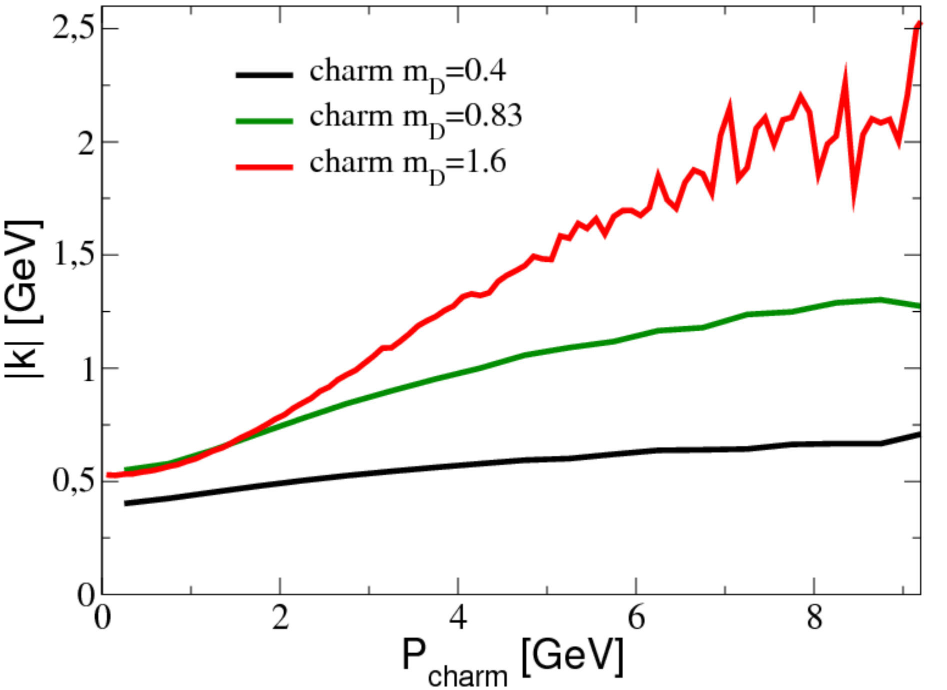
Boltzmann vs Langevin (Charm)

T=400 MeV

Angular dependence of σ



Mometum transfer vs P

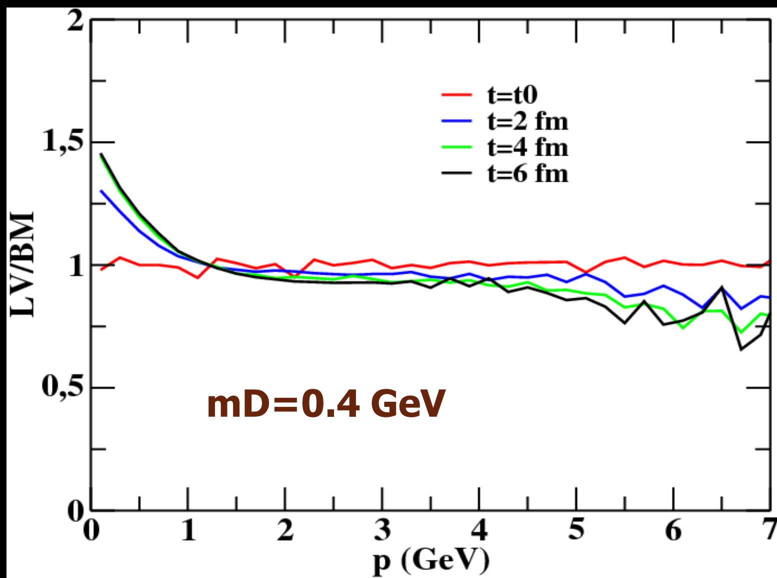


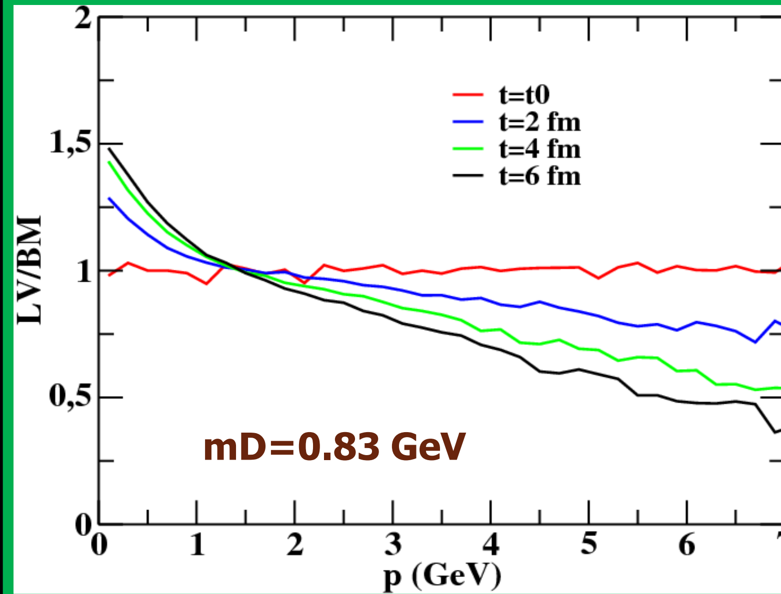
Decreasing m_D makes the σ more anisotropic ➡ **Smaller average momentum transfer**

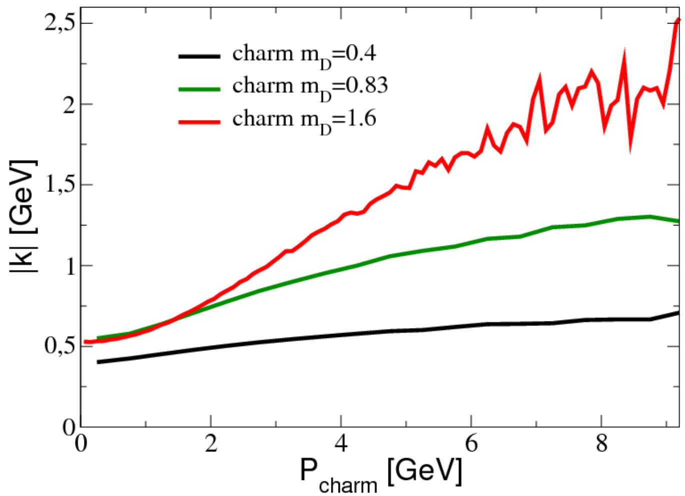
Hees,Greco,Rapp,PRC,73,034913 (2006)

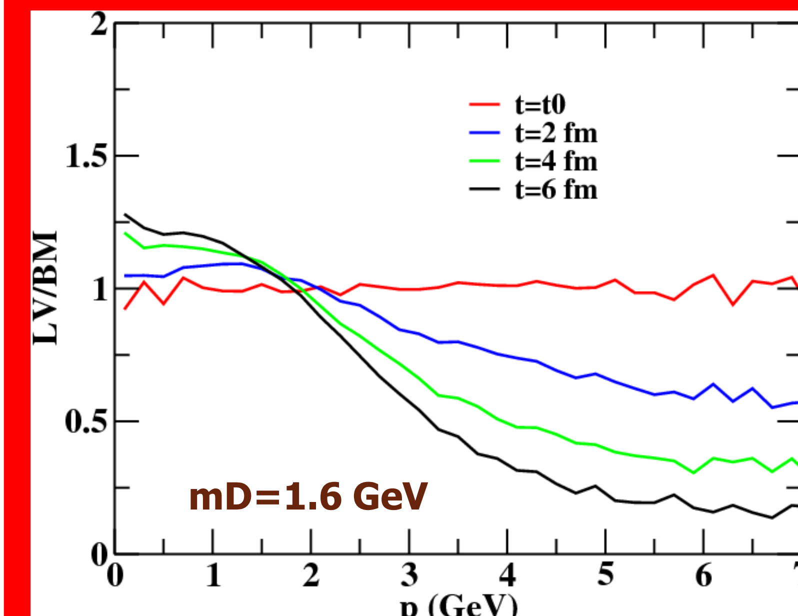
Das, Scardina, Plumari and Greco PRC,90,044901(2014)

Boltzmann vs Langevin (Charm)



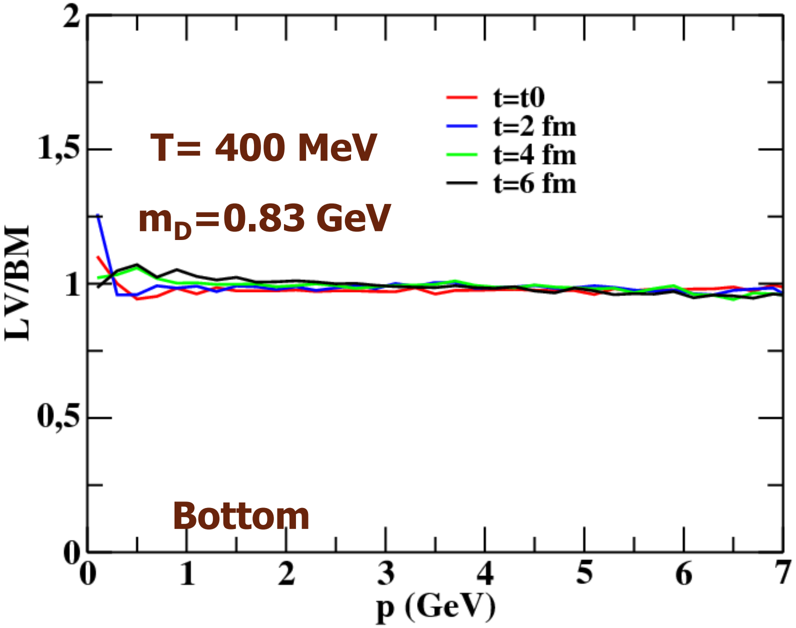


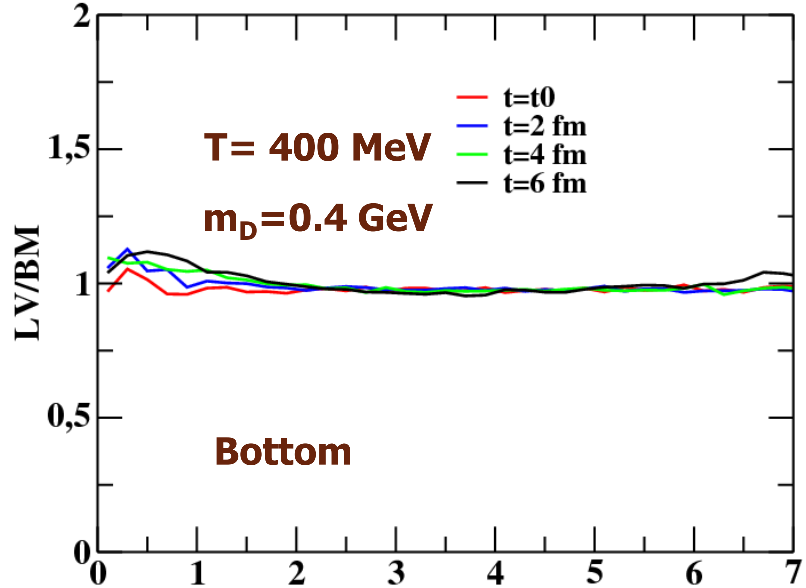


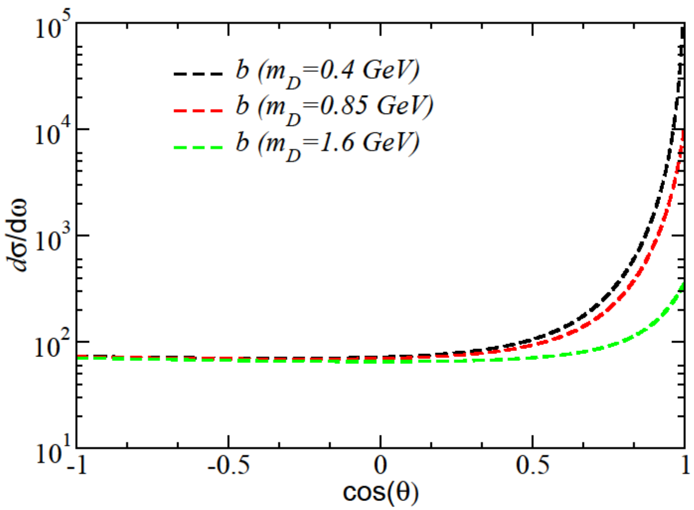


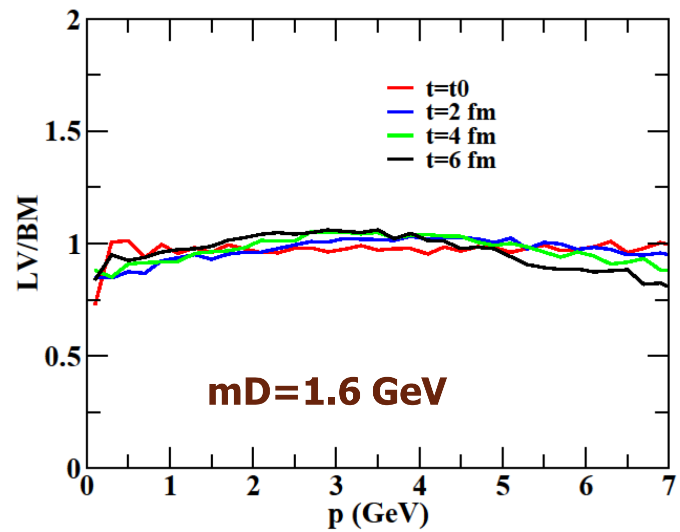
Das, Scardina, Plumari and Greco
PRC,90,044901(2014)

Bottom: Boltzmann = Langevin



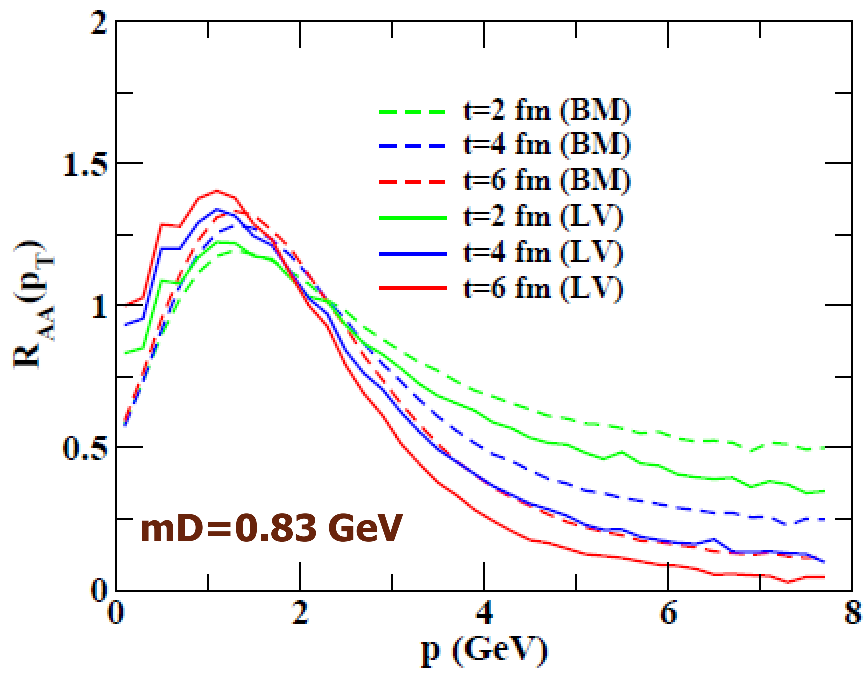


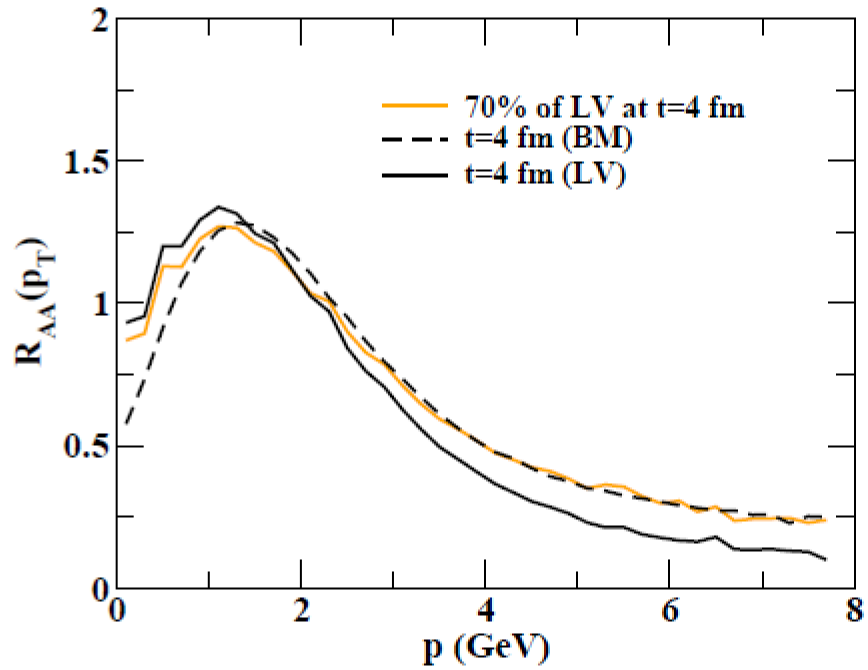




But Larger M_b/T (≈ 10) the better Langevin approximation works

Implication for observable, R_{AA}?





The Langevin approach indicates a smaller R_{AA} thus a larger suppression.

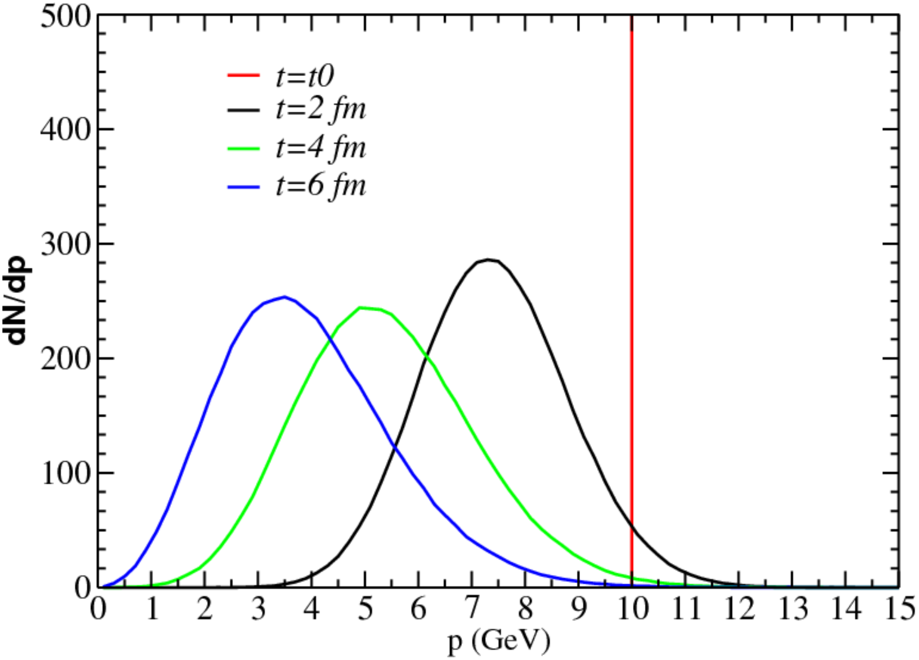
However one can mock the differences of the microscopic evolution and reproduce the same R_{AA} of Boltzmann equation just changing the diffusion coefficient by about a 30 %

Evolution: Boltzmann vs Langevin (Charm)

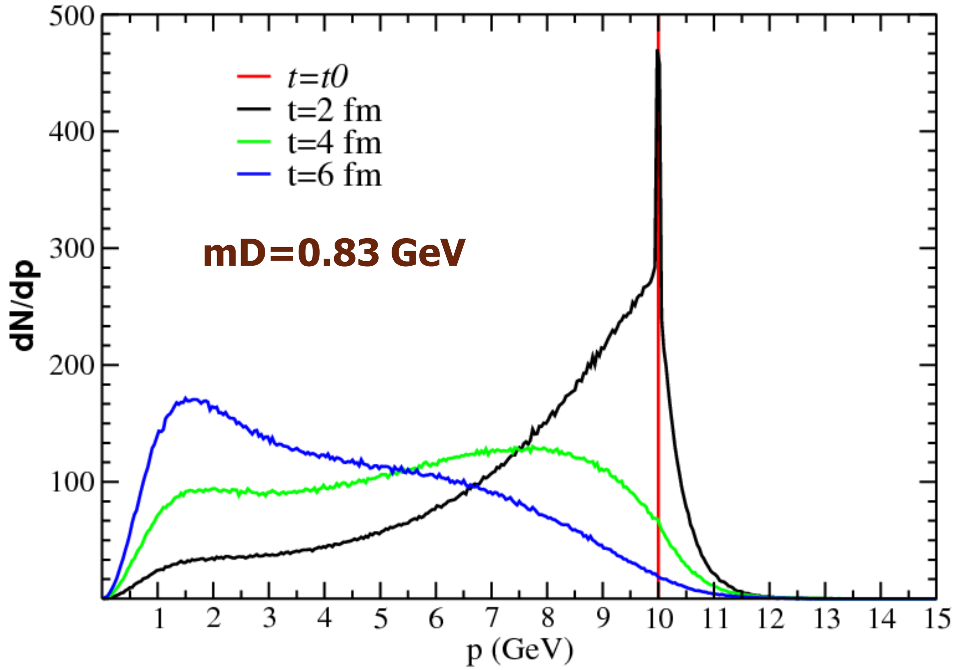
Momentum evolution starting from a δ (Charm) in a Box

$$\frac{dN}{d^3p}_{initial} = \delta(p - 10GeV)$$

Langevin



Boltzmann



In case of Langevin the distributions are Gaussian as expected by construction

In case of Boltzmann the charm quarks does not follow the Brownian motion

Das, Scardina, Plumari and Greco
PRC,90,044901(2014)

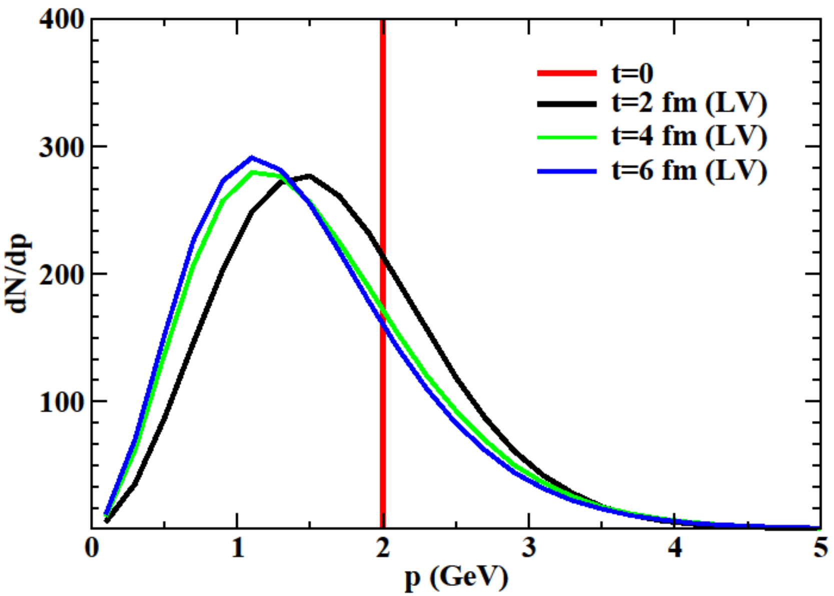
Evolution: Boltzmann vs Langevin (Charm)

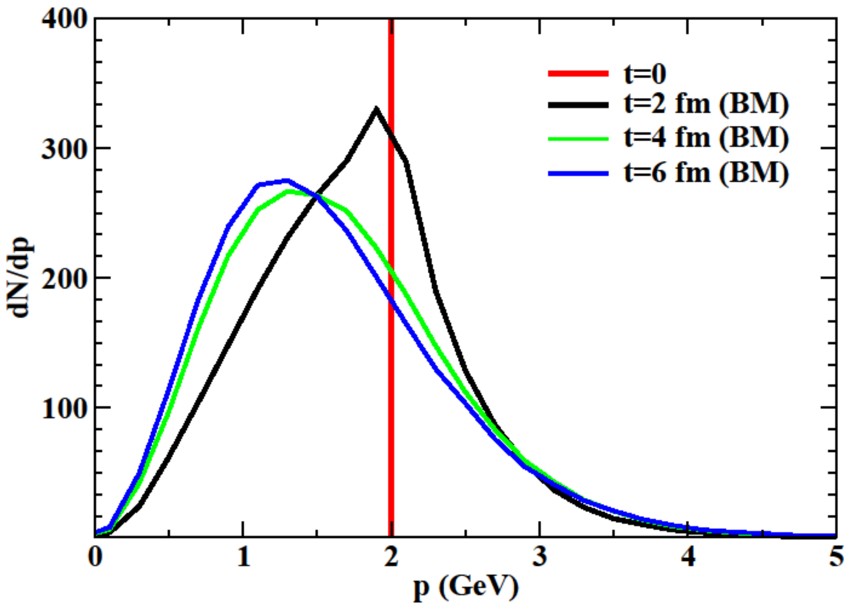
Momentum evolution starting from a δ (Charm) in a Box

$$\frac{dN}{d^3p}_{initial} = \delta(p - 2GeV)$$

Langevin

Boltzmann





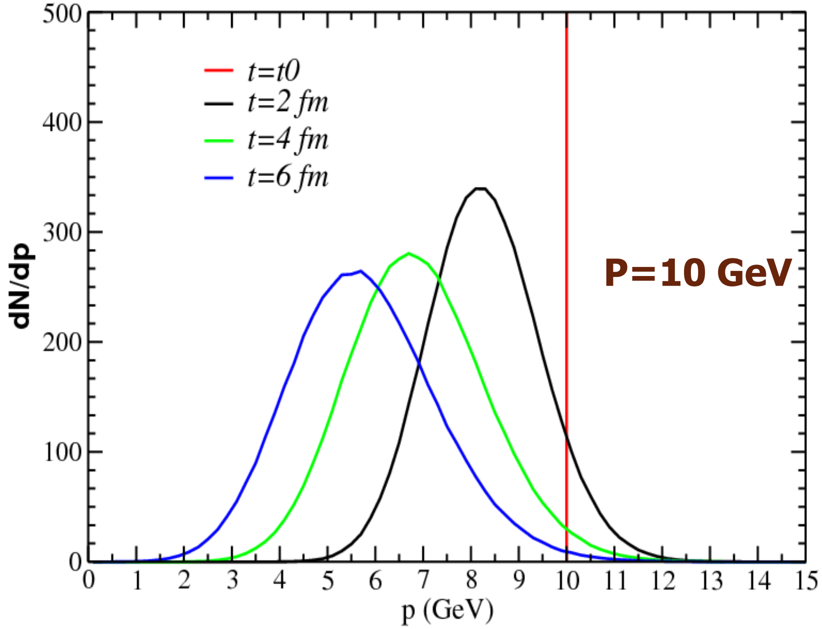
In case of Langevin the distributions are Gaussian as expected by construction

In case of Boltzmann the charm quarks follow the Brownian motion: At Low Momentum.

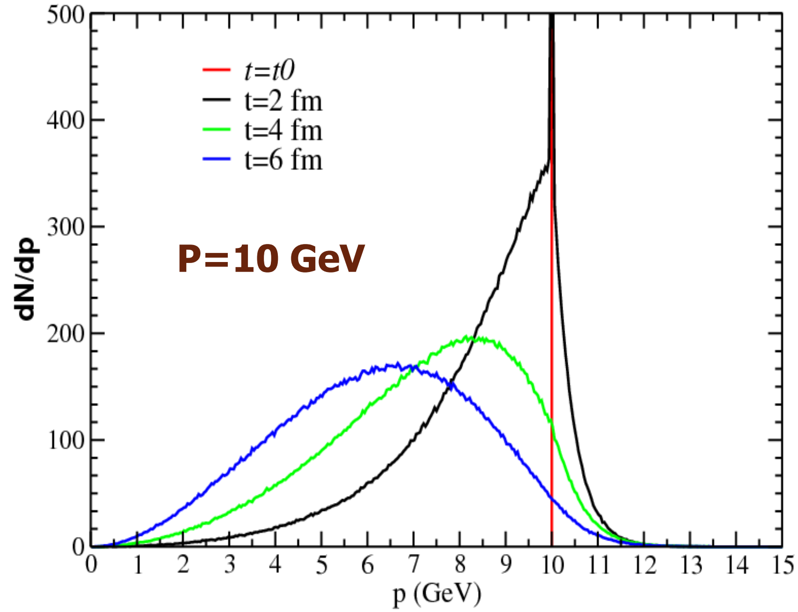
Momentum evolution starting from a δ (Bottom)

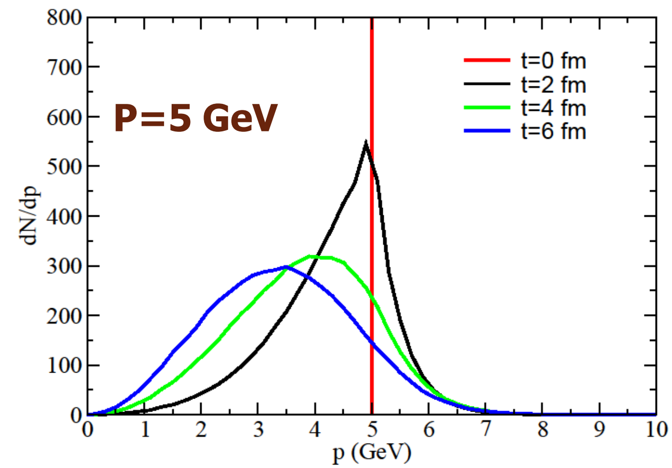
In a Box

Langevin



Boltzmann

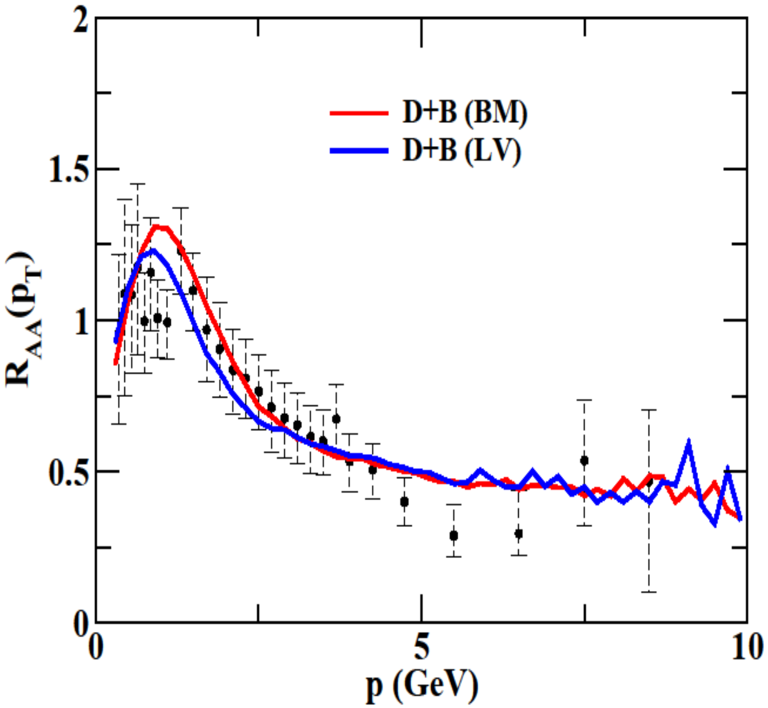


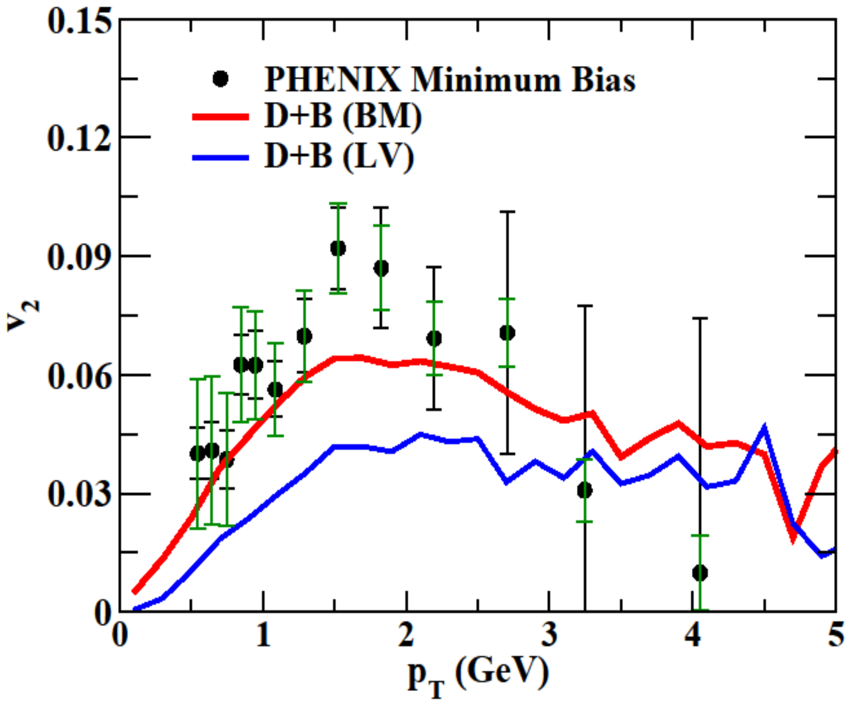


For bottom quarks it works better.

R_{AA} and v2 at RHIC

(With near isotropic cross-section)



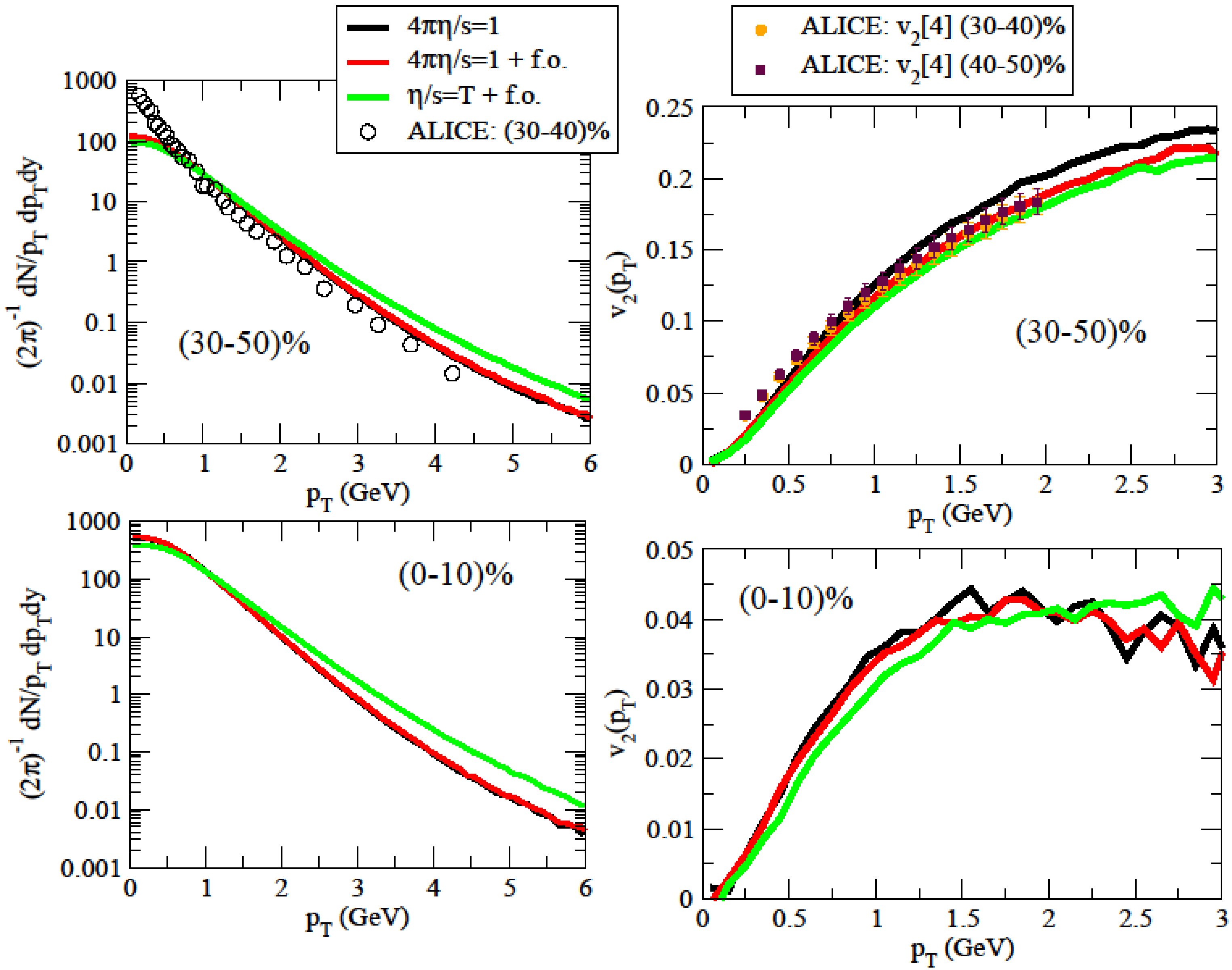


Das, Scardina, Plumari and Greco
PRC,90,044901(2014)

At fixed RAA Boltzmann approach generate larger v2 .
(depending on mD and M/T)

With isotropic cross section one can describe both RAA and V2 simultaneously within the Boltzmann approach !

Spectra and elliptic flow (RRTF assignments)



Different form of FDT

The long time solution is recovered relating the Drag and Diffusion coefficent by means of the fluctuation dissipation relation

We have studied the impact on R_{AA} and v_2 of evaluating the drag and the diffusion from pQCD or using the different form of the FDT

1) A and B_T (from pQCD No FDT, but $B_{||} = B_T$)

2) $B_{||} = B_T$ (pQCD) ; A (FDT) →

$$A = \frac{B_T}{ET} - \frac{1}{p}\frac{\partial B_T}{\partial p}$$

3) A pQCD ; D= $B_{||} = B_T$ (FDT) →

$$D = AET$$

Post-Ito

4) B_T and $B_{||}$ (pQCD) ; A FDT →

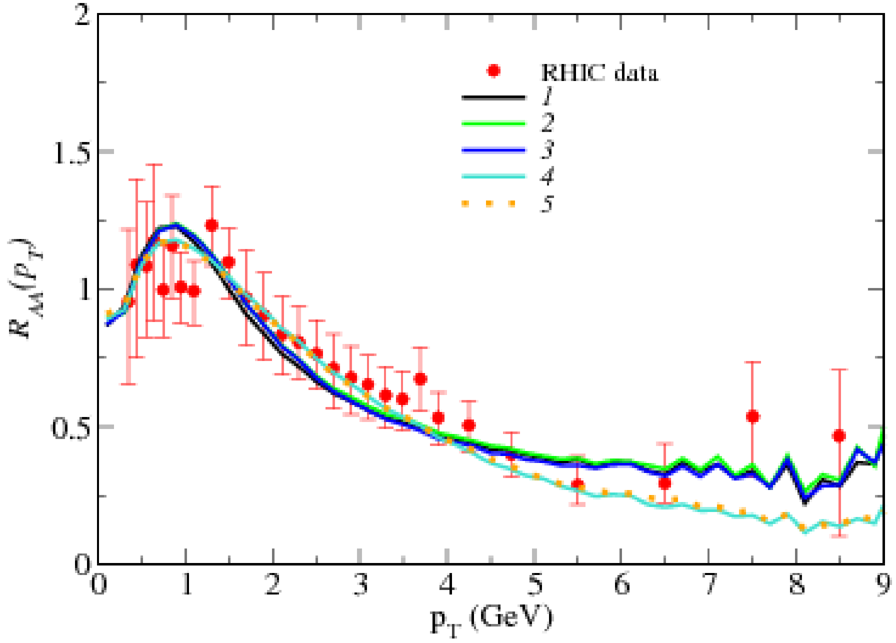
$$A = \frac{1}{p} B_{\parallel} \frac{1}{T}\frac{\partial E}{\partial p} - \frac{1}{p}\frac{\partial B_{\parallel}}{\partial p} - \frac{(n-1)}{p^2}\left(B_{\parallel} - B_{\perp}\right)$$

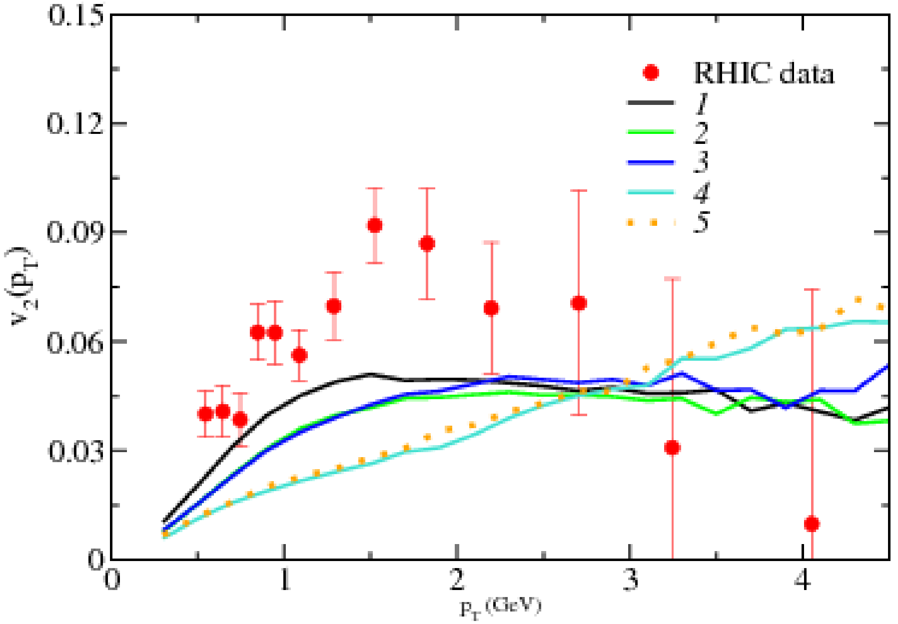
5) B_T=$B_{||}$ (pQCD) ; A FDT →

$$A = \frac{B_{\parallel}}{ET} - \frac{1}{p}\frac{\partial B_{\parallel}}{\partial p}$$

Impact on R_{AA} and v_2 of the different form of FDT

Au+Au (200 GeV) b=8.0





4) B_T and $B_{||}$ (pQCD) ; A FDT

m_D=gT

5) B_T=$B_|$ (pQCD) ; A FDT

If $B_{||}$ is evaluated from pQCD one has to reduce the drag and the diffusion by 55% but the p_T dependence of R_{AA} and v_2 is quite different